F1 LS 2023 Skúška – 2. termín
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(1.) $\dfrac{t, n, \omega = 2\omega_0}{\omega; \omega_0}$

$$\omega = 2\omega_0$$
$$2\omega_0 = \omega_0 + \varepsilon t$$
$$2\pi n = \omega_0 t + \tfrac{1}{2}\varepsilon t^2$$

$$\varepsilon = \frac{2\omega_0 - \omega_0}{t} = \frac{\omega_0}{t} \rightarrow 2\pi n = \omega_0 t + \frac{1}{2}\frac{\omega_0}{t}t^2$$

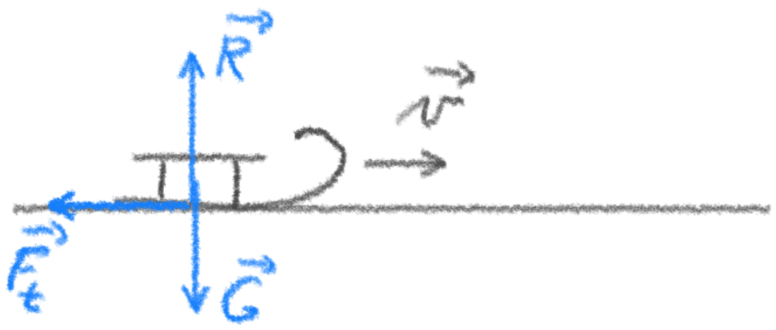
$$2\pi n = \omega_0 t + \tfrac{1}{2}\omega_0 t = \tfrac{3}{2}\omega_0 t \Rightarrow \underline{\underline{\omega_0 = \frac{4\pi n}{3t}}}$$

$$\underline{\underline{\omega = \frac{8\pi n}{3t}}}$$

(2.) $\dfrac{m, v_0, \mu}{s;}$

i) z energetickej bilancie

$F_t = \mu m g$



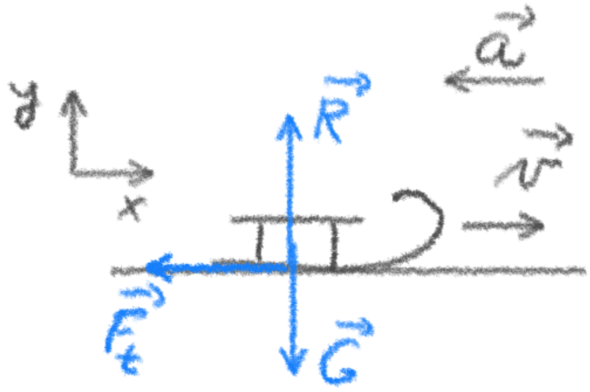
Práca trecej sily $W_t = \mu m g s = \Delta E_k$

zmena kinetickej energie $\Delta E_k = \frac{1}{2}m v_0^2 - 0$

$$\mu m g s = \frac{1}{2}m v_0^2 \Rightarrow \underline{\underline{s = \frac{v_0^2}{2\mu g}}}$$

ii) z rovnomerne spomaleného pohybu

$$m\vec{a} = \vec{G} + \vec{R} + \vec{F_t}$$



$$x: \quad -ma = -\mu R$$
$$y: \quad 0 = R - mg \Rightarrow R = mg$$

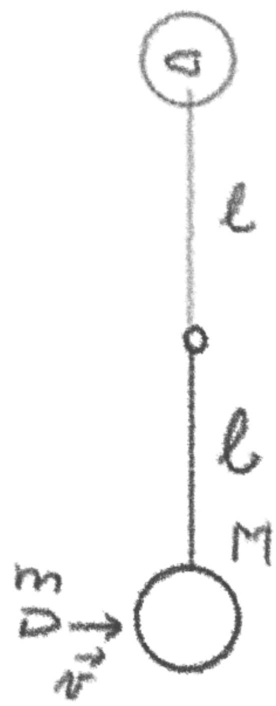
$$ma = \mu R = \mu mg \Rightarrow a = \mu g$$
$$s = v_0 t - \frac{1}{2} a t^2$$
$$0 = v_0 - at \Rightarrow t = \frac{v_0}{a}$$

$$s = v_0 \frac{v_0}{a} - \frac{1}{2} a \frac{v_0^2}{a^2} = \frac{v_0^2}{a} - \frac{1}{2} \frac{v_0^2}{a} = \frac{1}{2} \frac{v_0^2}{a}$$

$$s = \frac{1}{2} \frac{v_0^2}{\mu g}$$

3. m, M, l

$v;$



$$\frac{1}{2}(m+M) v_1^2 = (m+M) g 2l$$

$$v_1^2 = 4gl$$

$$mv = (m+M) v_1 \Rightarrow v_1 = \frac{m}{m+M} v$$

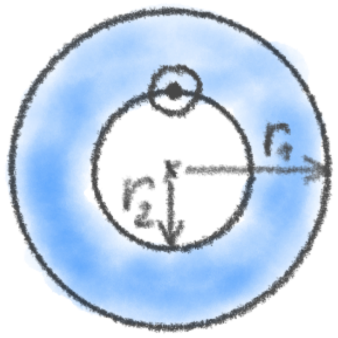
$$\frac{m^2}{(M+m)^2} v^2 = 4gl \Rightarrow v^2 = 4 \frac{(M+m)^2}{m^2} gl$$

$$v \geq \frac{2(M+m)}{m} \sqrt{gl}$$

4. $r_1;\ r_2 < r_1$

$T;$

$$T = 2\pi\sqrt{\frac{J}{mgr}}$$



$$r = r_2$$

$$J = J_{T1} - J_{T2} + mr^2$$

$$J_{T1} = \frac{1}{2} m_1 r_1^2 \qquad J_{T2} = \frac{1}{2} m_2 r_2^2$$

$$m_1 = \sigma \pi r_1^2$$

$$m_2 = \sigma \pi r_2^2 \qquad \sigma = \frac{m}{\pi(r_1^2 - r_2^2)}$$

$$J_{T1} = \frac{1}{2}\sigma\pi r_1^2 r_1^2 = \frac{1}{2}\frac{m r_1^4}{r_1^2 - r_2^2} \qquad J_{T2} = \frac{1}{2}\frac{m r_2^4}{r_1^2 - r_2^2}$$

$$J_{T1} - J_{T2} = \frac{1}{2}\frac{m}{r_1^2 - r_2^2}(r_1^4 - r_2^4) = \frac{1}{2} m \frac{(r_1^2 - r_2^2)(r_1^2 + r_2^2)}{r_1^2 - r_2^2} =$$

$$= \frac{1}{2} m (r_1^2 + r_2^2)$$

$$J = \frac{1}{2} m (r_1^2 + r_2^2) + m r_2^2 = \frac{1}{2} m (r_1^2 + 3r_2^2)$$

$$T = 2\pi\sqrt{\frac{m(r_1^2 + 3r_2^2)}{2mgr_2}} = \underline{\underline{2\pi\sqrt{\frac{r_1^2 + 3r_2^2}{2gr_2}}}}$$

5. $S_1,\ S_2 < S_1,\ v_1,\ p_2$

$v_2;\ p_1$

$$S_1 v_1 = S_2 v_2 \Rightarrow \quad \underline{\underline{v_2 = \frac{S_1}{S_2} v_1}}$$

$$\frac{1}{2}\rho v_1^2 + p_1 = \frac{1}{2}\rho v_2^2 + p_2$$

$$p_1 = p_2 + \frac{1}{2}\rho\left(v_2^2 - v_1^2\right) = p_2 + \frac{1}{2}\rho\left(\frac{S_1^2}{S_2^2} v_1^2 - v_1^2\right)$$

$$\underline{\underline{p_1 = p_2 + \frac{1}{2}\rho v_1^2\left(\frac{S_1^2}{S_2^2} - 1\right)}}$$

6. $R,\ \varphi = A + Bt^3,\ t_1$

$a_t,\ a_d,\ a$

$$a_t = R\varepsilon$$
$$a_d = R\omega^2$$

$$a = \sqrt{a_t^2 + a_d^2} = R\sqrt{\varepsilon^2 + \omega^4}$$

$$\omega = \frac{d\varphi}{dt} = \frac{d}{dt}\left(A + Bt^3\right) = 3Bt^2$$

$$\varepsilon = \frac{d\omega}{dt} = \frac{d}{dt} 3Bt^2 = 6Bt$$

$$a_t = R\varepsilon = \underline{\underline{6BRt_1}} \qquad a_d = R\omega^2 = \underline{\underline{9B^2Rt_1^4}}$$

$$a = \sqrt{36B^2R^2t_1^2 + 81B^4R^2t_1^8} = \underline{\underline{3BRt_1\sqrt{4 + 9B^2t_1^6}}}$$