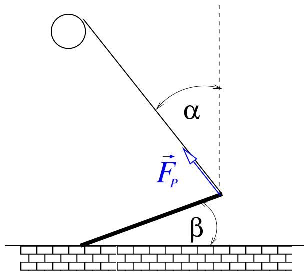
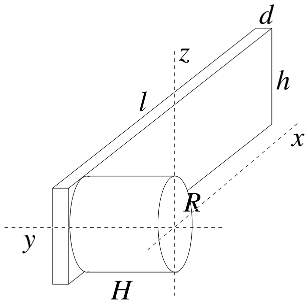
1. Dosku, pôvodne položenú na podlahe, dvíhame lanom uchyteným na jej pravom konci (Obr.). Lano je navíjané pevne umiestneným motorom. Na začiatku je $\beta_0 = 0$ a $\alpha_0 > 0$, no pretože koeficient statického trenia medzi ľavým koncom dosky a podlahou je len μ, doska sa pri počiatočných hodnotách α najprv prešmykuje, až neskôr sa pri hodnote $\alpha_c < \alpha_0$ zastaví. Aký bude uhol β_c v tomto okamihu?



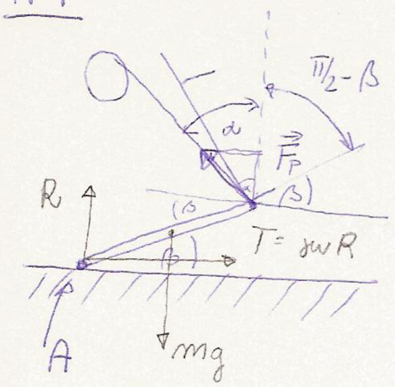
2. Nájdite tenzor zotrvačnosti ramena manipulátora s motorom vzhľadom na sústavu súradníc na obrázku. Rameno môžeme modelovať ako homogénu dosku s hmotnosťou m a s rozmermi h, l, a zanedbateľnou šírkou $d \approx 0$, a motor ako homogénny valec s výškou H, polomerom $R = 2h$ a hmotnosťou M. Vzorce pre tenzory zotrvačnosti valca (výška H, polomer R) a kvádra (hrany a, b, c) vzľadom na ich ťažiská a symetricky zorientované osi sú



$$\vec{\vec{I}}^{*}_{v} = \frac{M}{12}\begin{bmatrix} 3R^2+H^2 & 0 & 0 \\ 0 & 3R^2+H^2 & 0 \\ 0 & 0 & 6R^2 \end{bmatrix}, \quad \vec{\vec{I}}^{*}_{k} = \frac{m}{12}\begin{bmatrix} b^2+c^2 & 0 & 0 \\ 0 & a^2+c^2 & 0 \\ 0 & 0 & a^2+b^2 \end{bmatrix}.$$

Riešenia a bodovania sú na nasledovných stranách.

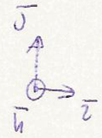
Pr 1



$$0 = \bar R + \bar T + \bar F_P + m\bar g \qquad \text{pre } \beta = \beta_c \text{ bude } T = \mu R \quad 1/2$$

$$\bar i: \quad \underset{1/2}{\mu R} - \underset{1/2}{F_P \sin\alpha} = 0 \qquad (1)$$

$$\bar j: \quad \underset{1/2}{R} + \underset{1/2}{F_P \cos\alpha} - \underset{1/2}{mg} = 0 \qquad (2)$$



moment síl vzhľadom na bod A na obrázku:

$$0 = \vec D = \underset{1/2}{\frac{l}{2} mg \sin\left(\beta + \frac{\pi}{2}\right)(-\vec k)} + \underset{1/2}{l F_P \sin\left(\alpha + \frac{\pi}{2} - \beta\right) \vec k}$$

$$0 = \frac{l}{2} mg \cos\beta\,(-1) + l F_P \cos(\alpha - \beta) \qquad (3)$$

$$(1) + (2) \rightarrow \quad \frac{F_P}{\mu} \sin\alpha + F_P \cos\alpha = mg$$

$$F_P = \frac{\mu mg}{\sin\alpha + \mu\cos\alpha} \qquad 1/2$$

dosadíme do (3)

$$0 = -\frac{\cancel{l}}{2} \cancel{mg} \cos\beta + \cancel{l} \frac{\mu \cancel{mg}}{\sin\alpha + \mu\cos\alpha} \cdot \cos(\alpha - \beta)$$

$$= -\frac{1}{2}\cos\beta + \frac{\mu}{\sin\alpha + \mu\cos\alpha}(\cos\alpha\cos\beta + \sin\alpha\sin\beta) \quad \Big| \frac{1}{\cos\beta} \, , \times \frac{\sin\alpha + \mu\cos\alpha}{\mu}$$

$$0 = -\frac{\cancel{\mu}(\sin\alpha + \mu\cos\alpha)}{2(\mu)} + \cos\alpha + \sin\alpha\,\mathrm{tg}\,\beta \quad \Big| \frac{1}{\sin\alpha}$$

$$0 = -\frac{1}{2\mu}(1 + \mu\,\mathrm{cotg}\,\alpha) + \mathrm{cotg}\,\alpha + \mathrm{tg}\,\beta$$

$$\mathrm{tg}\,\beta = \frac{1 + \mu\,\mathrm{ctg}\,\alpha}{2\mu} - \mathrm{ctg}\,\alpha = \frac{1 - \mu\,\mathrm{ctg}\,\alpha}{2\mu} \qquad 1/2$$

Pr 2 $\qquad R = h/2 \;!! \qquad h = 2R$

valec

$$\overline{\overline{I}}_v^* = \frac{M}{12}\begin{pmatrix} 3R^2+H^2 & 0 & 0 \\ 0 & 6R^2 & 0 \\ 0 & 0 & 3R^2+H^2 \end{pmatrix} \quad 1/2$$

posunutie $\bar{d}_v = \frac{H}{2}\vec{j}$ $\quad 1/2$

$$\Delta\overline{\overline{I}}_v = M\left(\bar{d}_v\bar{d}_v\overline{\overline{1}} - \bar{d}_v\bar{d}_v\right) = M\left(\frac{H^2}{4}(\bar{i}\bar{i}+\bar{j}\bar{j}+\bar{k}\bar{k}) - \frac{H^2}{4}\bar{j}\bar{j}\right) \quad 1/2$$

$$= M\left(\frac{H^2}{4}(\bar{i}\bar{i}+\bar{k}\bar{k})\right) = \frac{M}{12}\begin{pmatrix} 3H^2/4 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 3H^2/4 \end{pmatrix}$$

$$\overline{\overline{I}}_v = \overline{\overline{I}}_v^* + \Delta\overline{\overline{I}}_v = \frac{M}{12}\begin{pmatrix} 3R^2+4H^2 & 0 & 0 \\ 0 & 6R^2 & 0 \\ 0 & 0 & 3R^2+4H^2 \end{pmatrix} \quad (**) \quad 1/2$$

rameno:

$$\overline{\overline{I}}_r^* = \frac{m}{12}\begin{pmatrix} h^2 & 0 & 0 \\ 0 & \ell^2+h^2 & 0 \\ 0 & 0 & \ell^2 \end{pmatrix} \quad 1/2$$

posunutie

$$\vec{d}_r = H\bar{j} + \left(\frac{\ell}{2}-R\right)\bar{i} \quad 1/2$$

$$\Delta\overline{\overline{I}}_r = m\left(\bar{d}_r\cdot\bar{d}_r\,\overline{\overline{1}} - \bar{d}_r\bar{d}_r\right) = m\Big(\left(H^2+\left(\frac{\ell}{2}-R\right)^2\right)(\bar{i}\bar{i}+\bar{j}\bar{j}+\bar{k}\bar{k}) - H^2\bar{j}\bar{j} - \left(\frac{\ell}{2}-R\right)^2\bar{i}\bar{i}$$
$$- H\left(\frac{\ell}{2}-R\right)(\bar{i}\bar{j}+\bar{j}\bar{i})\Big) \quad 1/2$$

$$= m\left(H^2(\bar{i}\bar{i}+\vec{k}\vec{k}) + \left(\frac{\ell}{2}-R\right)^2(\bar{j}\bar{j}+\vec{k}\vec{k}) - H\left(\frac{\ell}{2}-R\right)(\bar{i}\bar{j}+\bar{j}\bar{i})\right) \quad 1/2$$

$$\overline{\overline{I}}_r = \overline{\overline{I}}_r^* + \Delta\overline{\overline{I}}_r = m\begin{pmatrix} h^2/12 + H^2 & -H\left(\frac{\ell}{2}-R\right) & \emptyset \\ -H\left(\frac{\ell}{2}-R\right) & \frac{\ell^2+h^2}{12} + \left(\frac{\ell}{2}-R\right)^2 & \emptyset \\ \emptyset & \emptyset & \frac{\ell^2}{12} + H^2 + \left(\frac{\ell}{2}-R\right)^2 \end{pmatrix} \quad (*) \quad 1/2$$

$$\overline{\overline{I}} = \underset{(*)}{\overline{\overline{I}}_r} + \underset{(**)}{\overline{\overline{I}}_v} \quad 1/2$$