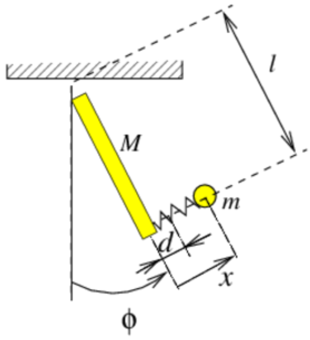
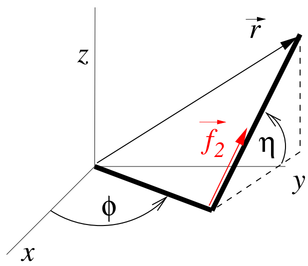
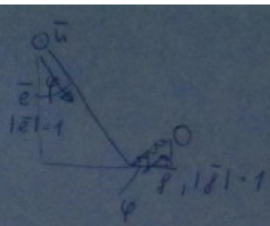
1. Nájdite Lagrangeovu funkciu a pohybové rovnice pre stupne voľnosti x a ϕ pre fyzikálne kyvadlo s guličkou upevnenou pomocou kolmej pružiny na jeho dolnom konci (obr.) Homogénna tyč má dĺžku l, hmotnosť M a moment zotrvačnosti okolo ťažiska I^*. Gulička má hmotnosť m, jej polomer považujte sa zanedbateľne malý. Pružina má tuhosť k a v uvoľnenom stave má dĺžku d.



2. Nájdite zložky koncového bodu manipulátora vzhľadom na osi x, y, z, ak dĺžka ramena zvierajúceho uhol ϕ s osou x je a, dĺžka ramena zvierajúceho uhol η s vodorovnou rovinou je b, a prvé a druhé rameno zvierajú medzi sebou pravý uhol. Aký uhol zviera jednotkový vektor $\vec{f_2}$ s osou x?



$L = E_k - W$ 1/2



$E_k = \frac{1}{2} m v_g^2 + \frac{1}{2} M v_t^2 + \frac{1}{2} \overset{*}{I} \omega_t^2$

1/4 1/4 1/4

$\omega_t = \dot{\varphi}$

$v_t = |\bar{v}_t| = \dots$ $\quad \bar{v}_t = \frac{d\bar{r}_t^*}{dt} = \frac{d}{dt} \frac{l}{2} \bar{e} = \frac{l}{2} \bar{\omega} \times \bar{e} = \frac{l}{2} \dot{\varphi}\, \bar{u} \times \bar{e}$

$|\bar{v}_t| = \frac{l}{2} |\dot{\varphi}|$ 1/4

$\bar{v}_g = \frac{d}{dt}(l\bar{e} + x\bar{f}) = l\bar{\omega} \times \bar{e} + \dot{x}\bar{f} + x\bar{\omega} \times \bar{f}$ 1/2

$= l\dot{\varphi} \underbrace{\bar{u} \times \bar{e}}_{\bar{f}} + \dot{x}\bar{f} + x\dot{\varphi} \underbrace{\bar{u} \times \bar{f}}_{-\bar{e}}$

$|\bar{v}_g|^2 = \bar{v}_g \cdot \bar{v}_g = (l\dot{\varphi} + \dot{x})^2 + x^2 \dot{\varphi}^2$ 1/4

$W = \overset{1/4}{-Mg\frac{l}{2}\cos\varphi} + \overbrace{\left(-l\cos\varphi + x\sin\varphi\right)mg}^{1/4} + \overbrace{\frac{1}{2}k(x-d)^2}^{1/4}$

$L = \frac{1}{2} m \left((l\dot{\varphi} + \dot{x})^2 + x^2\dot{\varphi}^2 \right) + \cancel{\dots}\, \overbrace{\frac{1}{2} M \frac{l^2}{4} \dot{\varphi}^2 + \frac{1}{2} \overset{*}{I} \dot{\varphi}^2}^{\frac{1}{2}\tilde{I}\dot{\varphi}^2}$

$+ Mg\frac{l}{2}\cos\varphi - (x\sin\varphi - l\cos\varphi)mg - \frac{1}{2}k(x-d)^2$

$x: \frac{\partial L}{\partial x} = m x \dot{\varphi}^2 - \sin\varphi\, mg - k(x-d)$ 1/4

$\frac{\partial L}{\partial \dot{x}} = m(l\dot{\varphi} + \dot{x})$ 1/4 $\quad \frac{d}{dt}\frac{\partial L}{\partial \dot{x}} = m(l\ddot{\varphi} + \ddot{x})$ 1/4

$\frac{d}{dt}\frac{\partial L}{\partial \dot{x}} - \frac{\partial L}{\partial x} = 0 \Rightarrow m(l\ddot{\varphi} + \ddot{x}) - m x \dot{\varphi}^2 + \sin\varphi\, mg + k(x-d) = 0$

1/2

$\varphi: \frac{\partial L}{\partial \varphi} = -Mg\frac{l}{2}\sin\varphi - (x\cos\varphi + l\sin\varphi)mg$ 1/4

$\frac{\partial L}{\partial \dot{\varphi}} = m\left((l\dot{\varphi} + \dot{x})l + x^2\dot{\varphi}\right) + \tilde{I}\dot{\varphi}$ 1/4 $, \quad \frac{d}{dt}\frac{\partial L}{\partial \dot{\varphi}} = m\left(l^2\ddot{\varphi} + l\ddot{x} + x^2\ddot{\varphi} + 2x\dot{x}\dot{\varphi}\right) + \tilde{I}\ddot{\varphi}$ 1/4

$\frac{d}{dt}\frac{\partial L}{\partial \dot{\varphi}} - \frac{\partial L}{\partial \varphi} = 0 \Rightarrow (m(l^2 + x^2) + \tilde{I})\ddot{\varphi} + m(l\ddot{x} + 2x\dot{x}\dot{\varphi}) = Mg\frac{l}{2}\sin\varphi + (x\cos\varphi - l\sin\varphi)mg$

1/4

Pr 2

$$\bar{r} = \bar{a} + \bar{b} \qquad \tfrac{1}{2}b$$

$$\bar{a} = \underset{\tfrac{1}{2}b}{a\bar{e}_1'} = a\, \mathcal{O}^{3,\varphi}[\bar{e}_1] = a \sum_{i=1}^{3} \bar{e}_i R_{i1}^{3,\varphi} = a(\bar{e}_1, \bar{e}_2, \bar{e}_3)\begin{pmatrix} c\varphi & \cancel{-s\varphi} & \cancel{0} \\ s\varphi & \cancel{c\varphi} & \cancel{0} \\ 0 & \cancel{0} & \cancel{1} \end{pmatrix} \qquad \tfrac{1}{4}b$$

$$= a(c\varphi\,\bar{e}_1 + s\varphi\,\bar{e}_2) \qquad \tfrac{1}{4}b$$

$$\bar{b} = \underset{\tfrac{1}{2}b}{b\bar{f}_2} = b\, \mathcal{O}^{1,\vartheta}[\bar{e}_2'] = b\sum_{i=1}^{3} \bar{e}_i' R_{i2}^{1,\vartheta}$$

$$= b \sum_{i=1}^{3}\sum_{j=1}^{3} \bar{e}_j \underbrace{R_{ji}^{3,\varphi}}_{1/4} \underbrace{R_{i2}^{1,\vartheta}}_{1/4}$$

$$= b(\bar{e}_1, \bar{e}_2, \bar{e}_3)\underbrace{\begin{pmatrix} c\varphi & -s\varphi & 0 \\ s\varphi & c\varphi & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{1/2}\underbrace{\begin{pmatrix} \cancel{1} & 0 & \cancel{0} \\ \cancel{0} & c\vartheta & \cancel{-s\vartheta} \\ \cancel{0} & s\vartheta & \cancel{c\vartheta} \end{pmatrix}}_{1/2}$$

1/2 - - - - - - - 1/2 . . . . . rotační matice

$$= b(\bar{e}_1, \bar{e}_2, \bar{e}_3)\begin{pmatrix} -s\varphi c\vartheta \\ c\varphi c\vartheta \\ s\vartheta \end{pmatrix}$$

1/4 . . . násobení matic o.k.

$$= b(-s\varphi c\vartheta\, \bar{e}_1 + c\varphi c\vartheta\, \bar{e}_2 + s\vartheta\, \bar{e}_3) \qquad \tfrac{1}{4}$$

úhel $\bar{e}_1$ a $\bar{f}_2$ . . . α

$$\bar{e}_1 \cdot \bar{f}_2 = \cos\alpha \qquad \tfrac{1}{2}$$

$$\bar{e}_1 \cdot \bar{f}_2 = -s\varphi c\vartheta = \cos\alpha \Rightarrow \underline{\underline{\alpha = a\cos(s\varphi c\vartheta)}} \qquad \tfrac{1}{2}$$