LANDAU-ZENER-STŰCKELBERG INTERFERENCE IN QUBIT-RESONATOR SYSTEM

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Received 04 May 2016; accepted 12 May 2016

1. Introduction

In this paper we theoretically study a system consisting of a quantum bit (qubit) coupled to a resonator in the limit of strong coupling and strong driving. This system can be described by a variety of theoretical approaches, however, only a few of them can be used in these limits. One such model is the adiabatic-impulse model [1], which is introduced here together with the obtained numerical results. Our approach was inspired by the experiment in [2], where a pattern similar to Landau-Zener-Stűckelberg interference was observed in a qubit-resonator system, leading to lasing in the resonator [3].

Adiabatic-impulse model was previously successfully used to describe dynamics of a single qubit, where it naturally leads to Landau-Zener-Stűckelberg interference and Rabi-like oscillations of qubits state. However, it was shown that in this model, the qubit cannot reach population inversion necessary for lasing without coupling to the environment. Here we show that in the scope of the adiabatic-impulse model, multiphoton processes in the qubit-resonator system naturally lead to an increase in the resonator photon number and nonthermal photon number distribution. We extended our model by introducing decoherence via the quantum-jump method, also known as Monte-Carlo wavefunction method [4].

2. Qubit –resonator system

Qubit-resonator system can be physically implemented as a superconducting flux qubit coupled to a coplanar waveguide resonator through mutual inductance or via shared Josephson junction [5]. The system can be controlled in two ways: either by changing the external magnetic flux threading the loop of the qubit or by transmitting microwave signal trough the resonator. In our case we consider the qubit to be driven by strong harmonic modulation of the external flux while no signal is being sent through the resonator. Hamiltonian of this system can be written as

$$H = H_{qb} + H_{res} + H_{\text{int}}, \tag{1}$$

$$H_{qb} = -\frac{1}{2}(\varepsilon\sigma_z + \Delta\sigma_x), H_{res} = \frac{1}{2}\omega_r a^\dagger a, H_{\text{int}} = g_k(\sigma^+ a^k + \sigma^- a^{\dagger k}), \tag{2}$$

where H_{qb}, H_{res} and H_{int} are the qubit, resonator and interaction Hamiltonian respectively. Operators a and $a^\dagger$ are creation and annihilation operators for a resonator, while σ_z and σ_x are Pauli operators acting on the qubit and $\sigma^\pm$ are qubit rising and lowering operators. Parameter Δ is energy gap of the qubit at the degeneracy point and ω_r is frequency of the resonator. Parameter ε is directly proportional to the driving magnetic field, which in case of harmonic modulation leads to

$$\varepsilon = \varepsilon_0 + A cos(\omega t). \quad (3)$$

The qubit-resonator coupling is considered to be small, $g_k \ll \Delta$. The interaction Hamiltonian in this form is known as multiphoton Jaynes-Cummings Hamiltonian [6].
If we neglect H_{int}, the Hamiltonian can be easily diagonalised for every t. Its eigenstates are denoted $|e/g\, ,n\rangle$, where e/gdenotes the excited/ground state of the qubit and $n$is the number of photons in the resonator. The eigenenergies of these states are $E_{e/g,n} = \pm\frac{\omega_{qb}}{2} + \omega_r\left(\frac{1}{2} + n\right)$, where $\omega_{qb} = \sqrt{\Delta^2 + \varepsilon^2}$. States $|e, n\rangle$and $|g, n + m\rangle$are degenerate at the points $\pm\varepsilon_m$where the equation $\sqrt{\Delta^2 + \varepsilon_m^2} = m\omega_r, m \in \mathbb{N}$is fulfilled. This equation has no solution for $m\omega_r < \Delta$. For simplicity, we chose$k$to be such, that $\omega_r > \Delta > (k - 1)\omega_r$,which is the lowest possible k for which there is no coupling between states that are not degenerate for some ε.

The interaction term substantially changes the eigenstates at degeneracy points $\varepsilon_k$only. There, the coupling will lift the degeneracy forming avoided energy crossings with the gap

$$g_{kn} = 2g_k\sqrt{\frac{(n+k)!}{n!}}. \quad (4)$$

Therefore, there are two kinds of avoided crossings in this system, qubit and qubit-resonatorcrossings. The qubit avoided crossings are at$\varepsilon = 0$ between pairs of states $|e, n\rangle$and $|g, n\rangle$ and are caused by coupling $-\frac{1}{2}\Delta\sigma_x$between qubit states. Therefore, the energy gap for every such crossing is Δ. The states $|\, e/g\, , n\rangle$are “adiabatic states” (eigenstates of the Hamiltonian with the coupling term $-\frac{1}{2}\Delta\sigma_x$included). The qubit-resonator avoided crossingsare at $\varepsilon = \pm\varepsilon_k$between pairs of states $|e, n\rangle$and $|g, n + k\rangle$ and are caused by coupling H_{int} between the qubit and the resonator. The gaps $g_{kn}$of these anticrossings depend on the photon number$n$. The states $|e, n\rangle$and $|g, n + k\rangle$are “diabatic” (eigenstates of the Hamiltonian with the coupling term H_{int} neglected).

Distinction between diabatic and adiabatic states is important for finding the correct form of time evolution matrix for Landau-Zener transitions at the crossings. It is worth noting, that adiabatic and diabatic states are approximately equal (up to their phases) far from the crossings, where coupling is negligible.

3. Adiabatic-impulse model

This model of time evolution of the system assumes that the evolution is adiabatic everywhere except close to avoided crossings, when instantaneous Landau-Zener transitions occur [1].
During the adiabatic evolution, the occupations of eigenstates stay constant and only their phases change. This evolution can be described by time evolution matrix of the form

$$U = \sum_{q,n} e^{-i\int E_{q,n}dt}|q, n\rangle\langle q, n|. \quad (5)$$

At avoided crossings, Landau-Zener transitions cause shifts in occupation probability of the levels as well as phase changes. For the qubit transitions at $\varepsilon = 0$, the transition matrix has the form

$$N = \sum_n \pm\sqrt{P}\,(|e, n\rangle\langle g, n| - |g, n\rangle\langle e, n|) + +\sqrt{1 - P}(e^{-i\Phi}|g, n\rangle\langle g, n| + e^{i\Phi}|e, n\rangle\langle e, n|)\,, \quad (6)$$

$$\delta = \frac{\Delta^2}{4|v|}\ , \quad (7)$$

$$\Phi = \frac{\pi}{4} + \delta(\ln\delta - 1) + \arg\Gamma(1 - i\delta)\,, \quad (8)$$

$$P = e^{-2\pi\delta}. \tag{9}$$

Here $v = \frac{d}{dt}\varepsilon\Big|_{\varepsilon=0}$, P is the so called Landau-Zener transition probability and ϕ is the Stokesphase obtained during the transition. The plus-minus sign depends on the sign of v, with $+$ for $v > 0$. During this transition, only the qubit's state changes.

For qubit-resonator transitions at $\varepsilon = \pm\varepsilon_k$, the transition matrix is

$$N' = \sum_{n\geq k} \pm\sqrt{1-P_n'}\,(e^{\pm i\,\phi_n'}|e,n\rangle\langle g,n+k| - e^{\mp i\,\phi_n'}|g,n+k\rangle\langle e,n|) + \\ +\sqrt{P_n'}(|g,n+k\rangle\langle g,n+k| + |e,n\rangle\langle e,n|)\,. \tag{10}$$

Here, $v_n' = \frac{d}{dt}\left(E_{g,n+k} - E_{e,n}\right)\Big|_{\varepsilon=\pm\varepsilon_k}$ is the slope of the energy difference between crossing states and P_n' and ϕ_n' are obtained from equations(7)-(9) , with Δ replaced by g_{kn} for every n.The plus-minus sign depends on the sign of v_n', with $+$ for $v_n' > 0$. During this transition, both state of the qubit and the photon number change.

Difference between matrices for qubit crossings and for qubit-resonator crossings is caused by distinction between diabatic and adiabatic states. It is worth noting that unlike in [1], the transition matrices in both diabatic and adiabatic basis depend on the direction of traversal, although for the single qubit case, derived results would be the same.

The overall time evolution of the system is given by the product of time evolution matrices for adiabatic parts and Landau-Zener transitions:

$$\psi(t) = \ldots N_2 U_2 N_1 U_1 \psi(0) \tag{11}$$

Visual representation of this model is very intuitive (Figure 1b). The system follows the adiabatic energy levels during adiabatic evolution and jumps between them during LZ transitions with the probability P. Probability amplitude of the system being in a given state is at every time given as the sum of amplitudes for different paths that lead to this state.

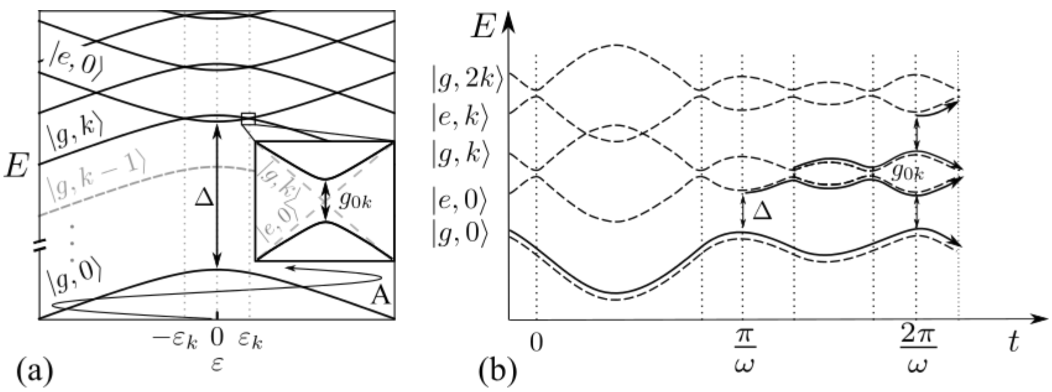

Figure 1: a) Energy levels of the qubit resonator system in dependence on $\boldsymbol{\varepsilon}$. *Adiabatic-impulse model predicts that while sweeping the* $\boldsymbol{\varepsilon}$, *occupation of the levels changes only at the avoided crossings, whose positions are marked by vertical dotted lines. This is depicted in b)**,** where dashed lines represent energy levels of the system, while thick solid lines represent various paths the system may choose during first period of its evolution, if it started in the ground state. Landau-Zener transitions are depicted as double headed arrows. Amplitude of the system being in any given state is given as the sum of amplitudes for different paths leading there.*

4. Quantum-jump method

This method was developed as a method to numerically solve the Lindblad master equation [4]. Within this approach, the system is always in pure state given by its wavefunction and decoherence takes the form of sudden jumps between the systems states. These jumps occur at random times and their frequency is given by decoherence rate. Therefore, between jumps, system evolves according to equation **(**11**)**, with only small addition to adiabatic phases caused by nonhermitian correction to Hamiltonian, which accounts for null-measurement. Evolution is simulated multiple times by this algorithm, and the resulting density matrix is given by averaging over all trials. Decoherence processes that were included in our model are qubit relaxation and resonator relaxation (via radiation of the photon to the connected microwave transmission lines). In our simple model, the decoherence rates were constant in time, i.e. they did not depend on $\boldsymbol{\varepsilon}$.

5. Numerical results

To obtain analytical results is difficult (although it can be done for a single qubit), but multiplication of matrices in equation (11) can be easily done numerically. In this paper, we present results for the following system parameters $\Delta = 2\pi \cdot 12.2$ GHz, $\omega = 2\pi \cdot 7.5$ GHz, $\omega_r = 2\pi \cdot 2.5$ GHz and $g_5 = 2\pi \cdot 0.05$ GHz, which were inspired by experiment in [3]. In this computation, only 20 resonator levels were taken into account, since for higher number of resonator levels, the average photon number did not change significantly. When the decoherence was included, we used qubit relaxation rate $\Gamma_{qb} = 2\pi \cdot 10^{-2} GHz$ and resonator relaxation rate $\Gamma_r = 2\pi \cdot 2 \cdot 10^{-3} GHz$.

In fig.2a we can see the number of photons in the resonator averaged over 10000 driving periods in dependence on driving offset ε_0 and amplitude A. We see that there are areas where number of photons in the resonator is increased. As seen fig. 2b, photon number distribution in these areas is non-thermal even if a decoherence is included.

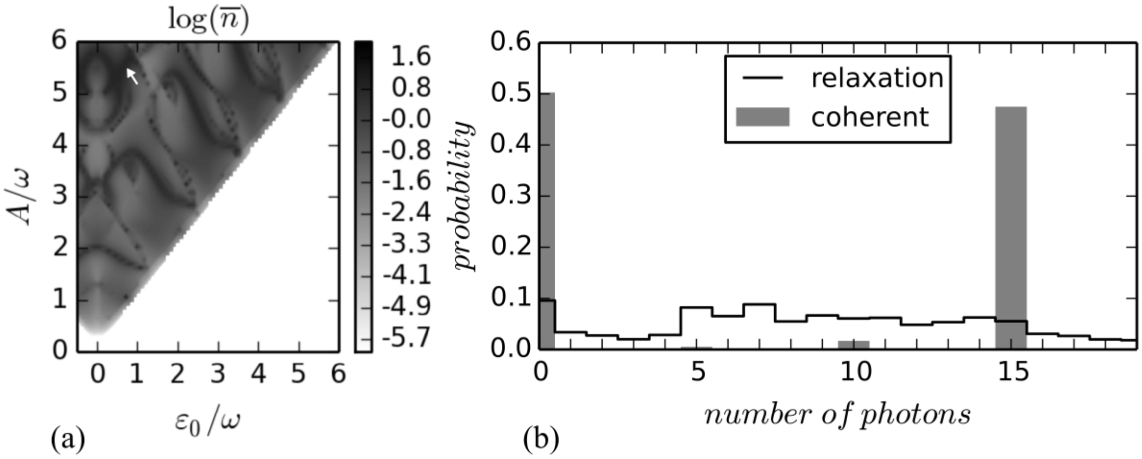

Figure 2:a) Logarithm of average number of photons in the resonator in dependence on driving parameters. No decoherence was included. Areas with substantially higher average photon number form pattern similar, but different from Landau-Zener-Stűckelberg interference pattern in a single qubit. In our range of parameters, the maximal average number of photons was 7.3 (white arrow). b) Probability distribution of photon number in maximum. In coherent case, system can only get to states with $n = mk$ $(m \in \mathbb{N}, k = 5)$ photons. Inclusion of the relaxation (with other parameters unchanged) enabled the system to access other states. Relaxation thus paradoxically caused increase in average photon number to 8.6.

6. Conclusion

We developed a model describing a multilevel system of a qubit strongly coupled to a resonator under strong driving in the framework of Landau-Zener-Stűckelberg interference. Our model predicts non-thermal probability distribution of photon number in areas with high average photon number. The role of decoherence in the system is nontrivial. Higher decoherence of qubit leads to higher average photon numbers. This can be explained by the fact, that by radiating photon from the resonator, system can reach states outside of the set $n = mk, m \in \mathbb{N}$, thus enabling more paths by which the system can ascend to the higher states.

Acknowledgement

The research leading to these results has received funding from the European Community's Seventh Framework Programme (FP7/2007-2013) under Grant No. 270843(iQIT) and DO7RP-0032-11. This work was also supported by the Slovak Research and Development Agency under the contracts APVV-14-0605, APVV-0808-12 (former projects No. APVV-0515-10). The authors gratefully acknowledge the financial support of the EU through the ERDF OP R&D, Project CE QUTE &metaQUTE, ITMS: 24240120032 and CE SAS QUTE.

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