NUMERICAL ANALYSIS OF FIRST-ORDER TEMPORAL SOLITONS

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Received 30 April 2016; accepted 12 May 2016

1. Introduction

In general, the temporal and spectral shape of a short optical pulse changes during propagation in a transparent medium due to self-phase modulation and chromatic dispersion. The most remarkable fact is actually not the possibility of the balance of dispersion and nonlinearity, but rather that soliton solutions of the nonlinear wave equation are very stable.

Solitons are also very stable against changes of the properties of the medium, provided that these changes occur over distances which are long compared with the soliton period. Soliton period is defined as the propagation distance in which the constant phase delay is π/4. Fundamental soliton pulses are technically very important for long-distance optical fiber communications and also in mode-locked lasers [1].

Classes of solitons:

a) **Bright temporal envelope solitons:** Pulses of light with a certain shape and energy that can propagate over large distances.
b) **Dark temporal envelope solitons:** Pulses of darkness within a continuous wave, where the pulses are of a certain shape, and possess propagation properties similar to the bright solitons.
c) **Spatial solitons:** Continuous wave beams or pulses, with a transverse extent of the beam that via the refractive index changes due to optical Kerr-effect can compensate for the diffraction of the beam.

In this paper we have focused on the bright and dark temporal soliton and their propagation in Kerr type medium [2].

2. Temporal analysis

The GVD (Group velocity dispersion) broadens optical pulses during their propagation inside an optical fiber. These pulses can be initially chirped or chirp can be generated inside the pulse during propagation. More specifically, a chirped pulse can be compressed during the early stage of propagation depending on the sings of chirp parameter C and the the GVD parameter β_2. Since $\beta_2 < 0$ in the 1.55 μm wavelength region of silica fibers, the condition $\beta_2 C < 0$ is satisfied. SPM- induced chirp is power dependent so we can imagine that under certain condition the SPM- induced chirp can cancel the GVD- induced broadening of the pulse. The optical pulse would then propagate undistorted in the form of a soliton [3].

In the introduction of this paper we have mentioned the soliton period term. The soliton period Z_0 and soliton order N play important role for identification of temporal solitons.

$$Z_0 = \frac{\pi}{2} L_D = \frac{\pi}{2} \frac{T_0^2}{|\beta_2|}, \quad (1)$$

where the L_D denotes the dispersion length and T_0 represents the pulse width. The effect of dispersion is representing by the GVD parameter β_2. Temporal solitons are attractive for optical communications because they are able to maintain their width even in presence of fiber dispersion. Only a fundamental soliton maintains its shape and remains chirp-free during propagation inside optical fibers.

For the paraxial approximation, the (1+1)/D variable coefficient GNLSE (generalized nonlinear Schrodinger equation [4]) has form

$$i\frac{\partial u}{\partial z} = -\frac{\beta_2(z)}{2}\frac{\partial^2}{\partial t^2} - \gamma(z)|u|^2 u - [v(z,t) + iw(z,t)]u \quad (2)$$

where $u(z,x)$ is the complex envelope of the electrical field, t is the time and z represents longitudinal propagation coordinate. The variable coefficient $\gamma(z)$ is the nonlinear parameter, $v(z,t)$ and $w(z,t)$ are the real and imaginary parts of the complex field. In order to find the soliton solution from Eq. (2) we must the transformation defined as fallows

$$u(z,t) = C(z)^{3/2}(\xi,\tau)\exp[i\varphi(z,t)], \quad (3)$$

here$C(z)$ is the temporal chirp function, which is related to $\beta_2(z)$ and C_0 is the initial chirp value.

3. Dark soliton

Firs we must mentioned that there are two basic types of dark solitons, one is called Black soliton and the second type is called Gray soliton. The numerical method of lines [5] can be employed to analyse the generation of dark solitons for any shape of the input beam profile. We have described the properties of the temporal dark solitons in (1+1)- dimension geometry. The simple example consider an input beam in the form

$$u(0,t) = u_0 \tanh(at), \quad (4)$$

Where the ratio $u_0/a$is arbitrary [6]. If the intensity of the input beam does not vanish at any point, the black soliton is not generated.

4. Numerical results

By numerical solving the GNLS equation (2) we can simulate the propagation of first order temporal soliton in standard SM fiber optic cable (G. 652). For generating the first order temporal soliton a hyperbolic cosine type of pulse was used. The initial chirp function represent by chirp parameter C was initially set to one and also the time step was set to by very small by comparing to pulse width. Initial pulse width T_0=150ps and the final shape of soliton pulse was moving in time by computing using the time step k=20ps. Fig.1 consist of many identical first order soliton pulse shapes captured in different times. This was only a first step in our investigation. After this we can start with the including also the space movement in z direction.

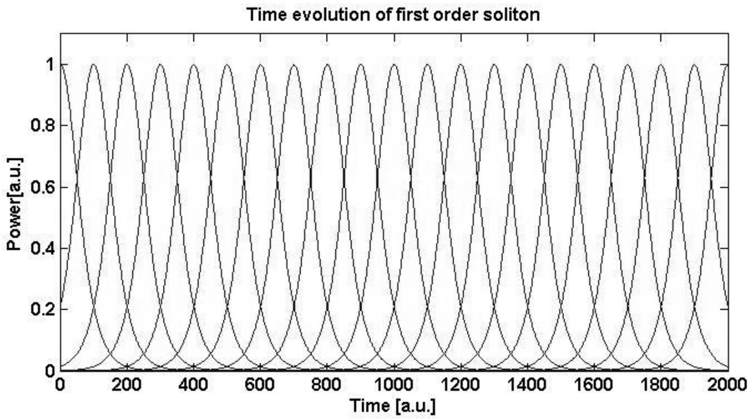

Fig.1:*The propagation of first order temporal soliton in 2-D distribution*

The second step of our investigation includes the space-time step and this problem was solved by using the time vector moving in space direction. Distance was calculated as a ratio of distance z and the calculated dispersion length L_D. Inside of a cycle of program the z was calculated as the soliton period using the equation (1). As result of this program were the space-time movement of first order soliton generated periodically depending on the value of soliton period and also on the value of dispersion and nonlinear parameter. From Fig. 2 we can observe at the special distance the full compensating of dispersion effects by the self-phase modulation represented by nonlinear parameter .

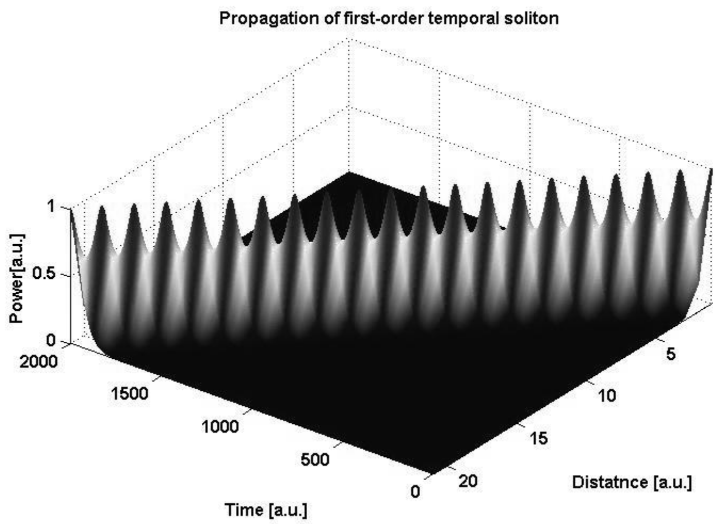

Fig.2: 3-D plot of first order temporal soliton in relation of distance shift. Dispersion parameter β_2=20ps/nm/km and the nonlinear parameter γ= 0.5W^{-1}/km

For generating the dark soliton pulse there were selected the hyperbolic secant type input pulses. Using numerical method of Lines we can numerically solved the GNSL equation (2) by using the input pulse shape in form

$$u(0,t) = u_0[1 - N\operatorname{sech}^2(at)]. \quad (5)$$

Fig. 3 shows characteristic example of a single black soliton in case of input beam in form of Eq. (5) with values *a* and *N* described below the figure. Depending on value of *N*we can generate first and also high-order solitons. The value $u_0 \sin \varphi$ has the meaning of the soliton velocity in space direction. In case when the $\varphi=0$ we can generate the so-called fundamental dark soliton as we can observe in Fig. 3.

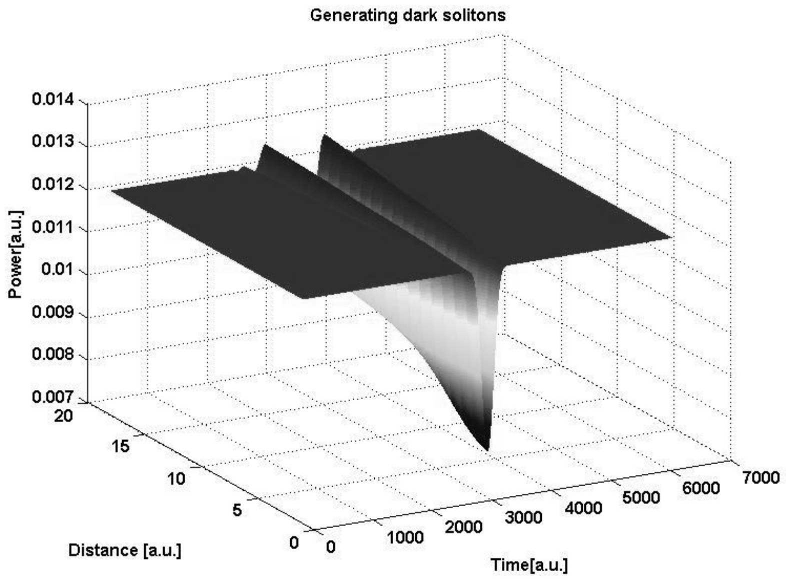

Fig.3:*Generation of fundamental dark soliton using input beams of the form of equation (5) with parameters a=0.9, N=1 and* $u_0=1$

When the $\cos^2 \varphi < 1$, this case corresponds to the so-called gray solitons. As we can observe in Fig. 4 only pair of grey solitons are generated. Bright solitons have a constant phase across the localized region but the dark solitons have a nontrivial distribution of their phase.

For some nonlinear functions $\gamma(|u^2|)$, dark solitons solutions of the GNLS (2) can be found in an explicit analytical form. There exist only a few cases (two) types of dark solitary waves in the model of cubic-nonlinearity. One of the crucial things is the soliton stability by propagating on optical fiber. Fig. 4 describes the gray soliton wave in the SM optical fiber with dispersion parameter value β_2=20 ps/km/nm.

One of the options of stabilization this type of solitons waves can be done by stabilization with nonlinear gain. The main goal of this paper was to create conditions when the both of dark solitons types were generated and also the purpose was to investigate the stability criteria. Investigation of soliton stability or instability was the main goal of my previous work in paper reference [7].

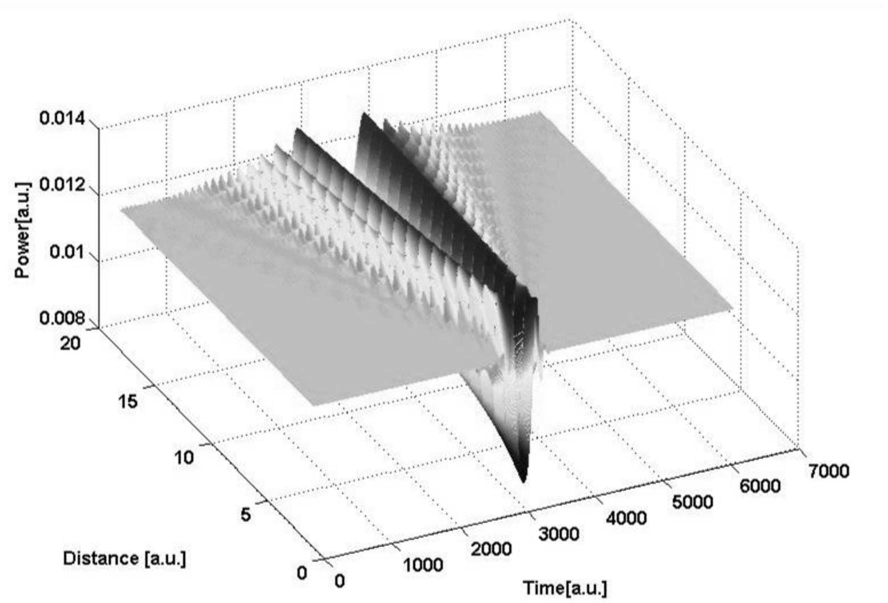

Fig.4:*Generation of gray soliton using input beams of the form of equation (4) with parameters a=0.9,N=1.5 and* $u_0=1$

The future work is going to be focused on the investigation of stability criteria in case of dark soliton, especially in case of black and gray solitons. In this we have to demonstrate only the fundamental solitons types and that is also one of the future work points to generate high-order dark solitons types.

Acknowledgement

This work was partly supported by the Slovak Research and Development Agency under the project No. APVV-0025-12, by Scientific Grant Agency of the Ministry of Education, Science, Research and Sports of the Slovak Republic.

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