THIN LAYER ORIENTED MAGNETOSTATIC CALCULATION MODULE FOR ELMER FEM, BASED ON THE METHOD OF THE MOMENTS

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1. Introduction

Effective and accurate modelling is the most important factor of successful development of different types of sensors or mechatronics devices [1]. For this reason, methods of modelling of different systems has been intensively developed during last fifty years. Additional factor, which gives the fresh impetus to the development of modelling techniques is the Moore' Law [2], describing exponential grow of computing power available for modelling.

Among different methods of modelling, the finite element method [3] is one of the most important. Application of this method creates the unique possibility of understanding of different physical systems described by the differential equations. This covers mechanical systems, electrical circuits, magnetostatic and magnetodynamic systems covering microwaves as well as flows.

Due to the strong commercial potential, proprietary software for finite element method is extremely expensive. For example, during last years, Polish research institutionswereunable to buy commercial licence for finite element method-based modelling of microwave chambers for industry-oriented purposes.

These barriers disappeared due to introduction of open source ELMER FEM software [4] enabling solving of all types of differential equation based systems, including mechanical, flow, magnetodynamic and microwave. This software, freely available for all, private and public, research and development bodies creates new possibilities of both fundamental research as well as development of sensors or unique measuring equipment, such as microwave moisture analyser.

However, finite element method has significant weakness connected with modelling of the thin layers [5], where thickness is important. In such a case, thin layer can't be reduced to 2D system. Such problem appear in the case of modelling of magnetostaticsystems covering thin layers. Moreover, since the commercial introduction of amorphous metals [6], thin films and layers are commonly used in development of different magnetic sensors. As a result, lack of possibility of modelling the magnetic sensors consisting of thin films (such as fluxgates or magnetoresistive sensors) is the significant barrier, which should be overcame.

2. Limitations of finite elements method for thin films

Meshing is the key operation for finite element modelling process. The most commonly, the tetrahedral elements (called the first order elements) are used for meshing. There are different meshing algorithms, however, for magnetostatic calculations (e.g. based on Whitney edge elements), tetrahedral elements should be close to regular tetrahedron. It means, that all edges of tetrahedral element should be similar. Significant differences in the length leads to the numerical inaccuracies in calculations.

Figure 1 presents the adaptive tetrahedral meshes created by the tetrahedral elements close to the regular tetrahedron. Meshes were generated by NETGEN open-source software

[7] . It can be easily observed, that decrease of relation between the thickness of the element and its length leads to very fast increase of the number of elements.

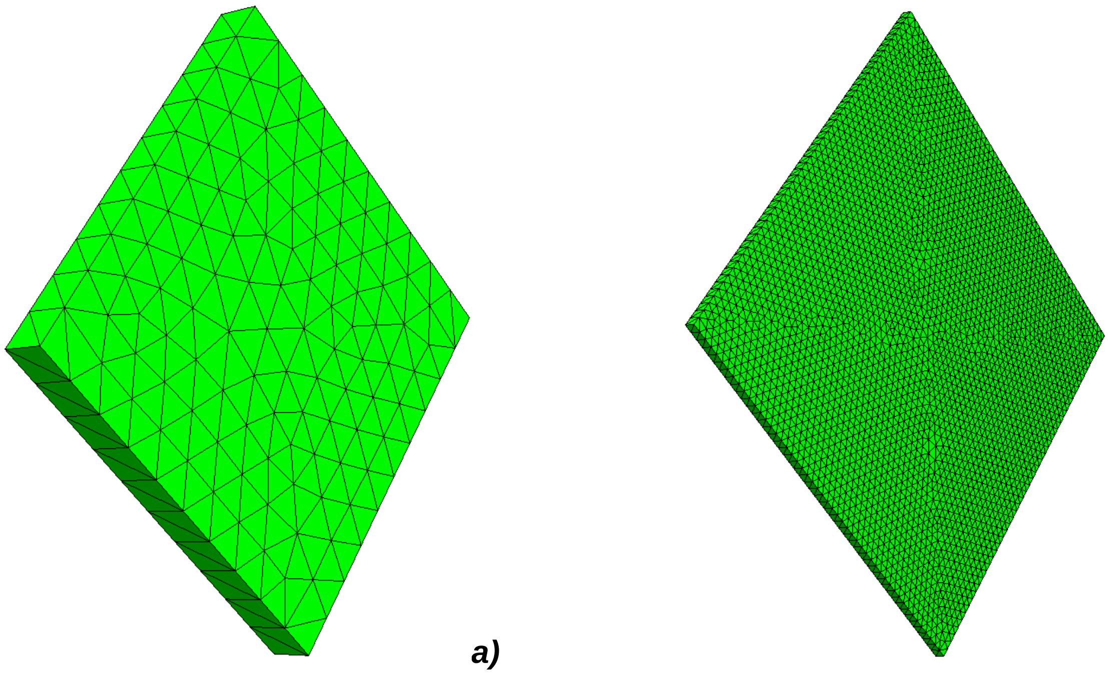

Fig.1:*Adaptive mesh made of by the tetrahedral elements close to the regular tetrahedron for elements with different length to thickness ratio k: a) 10, b) 50 .*

For tetrahedral elements close to the regular tetrahedron, number of elements *N* can be easily estimated from the following equation:

$$N = 1.2\, k^2 \tag{1}$$

where k is length to thickness ratio of the element. It means, that for the magnetostatic element made of amorphous alloy, with 1 cm length and thickness equal to 30 μm, where k=500, about N=100 000 elements should be considered. Due to the fact, that finite elements method requires solving of ill-posed differential equations due to conjugate gradient optimisation methods, such set of equations is very difficult to be solved from the point of view of computation time and memory resources.

3. Principles of the method of the moments

Method of the moments (MoM) is the method of magnetostatic calculations [8], which can be considered as the alternative for finite elements method. In the method of the moment, demagnetization effects are connected with the magnetic moments assigned to the each unit cell of the modelled body. However, in the case of MoM, the uniform, cubic element based meshes of 3D elements are the most suitable.

In the case of thin layers, 3D cubic elements are reduced to prisms, with height equal to the thickness of the layer. As it is presented elsewhere, the magnetizations $M_x\left(k_x, k_y\right)$ and $M_y\left(k_x, k_y\right)$ of the prism-shaped cell, which location is given by numbers $\left(k_x, k_y\right)$, are given by the following equations [9]:

$$M_x(k_x,k_y) + (\mu(k_x,k_y)-1)\cdot c \cdot \sum_{iy=1}^{n}\sum_{ix=1}^{n} M_x(i_x,i_y)\cdot\left(c_{xx}(i_x,i_y,k_x,k_y,\Delta L) - c_{xx}(i_x-1,i_y,k_x,k_y,\Delta L)\right) + $$
$$+(\mu(k_x,k_y)-1)\cdot c\cdot \sum_{ix=1}^{n}\sum_{iy=1}^{n} M_y(i_x,i_y)\cdot\left(c_{xy}(i_x,i_y,k_x,k_y,\Delta L) - c_{xy}(i_x,i_y-1,k_x,k_y,\Delta L)\right) = \mu(k_x,k_y)\cdot H_{x\,ext} \quad (2)$$

$$M_y(k_x,k_y) + (\mu(k_x,k_y)-1)\cdot c \cdot \sum_{ix=1}^{n}\sum_{iy=1}^{n} \left[M_y(i_x,i_y)\cdot\left(c_{yy}(i_x,i_y,k_x,k_y,\Delta L) - c_{yy}(i_x,i_y-1,k_x,k_y,\Delta L)\right)\right] + $$
$$+(\mu(k_x,k_y)-1)\cdot c\cdot \sum_{iy=1}^{n}\sum_{ix=1}^{n} \left[M_x(i_x,i_y)\cdot\left(c_{yx}(i_x,i_y,k_x,k_y,\Delta L) - c_{yx}(i_x-1,i_y,k_x,k_y,\Delta L)\right)\right] = \mu(k_x,k_y)\cdot H_{y\,ext} \quad (3)$$

where c_{xx}, c_{xy}, c_{yx}, c_{yy} are coefficients describing relative location of interacting cells, ΔL is cell's length, and $\mu(k_x, k_y)$ is the relative permeability of the material in considered cell, as well as $H_{y\,ext}$ and $H_{y\,ext}$ are values of external magnetizing field strength.

Approach to magnetostatic simulations, provided by the method of the moments, gives two following advantages:

- number of prism-shaped cells is not depending on the layer's thickness,
- values of magnetization $M_x(k_x,k_y)$ and $M_y(k_x,k_y)$ of the prism-shaped cell are calculated from the well-defined set of linear equations,

As a result, in the case of magnetostatic simulations in thin layers, MoM approach is much more effective, than finite elements method.

Solver, suitable for MoM was implemented using open-source OCTAVE 4.0 [10] software, which is MATLAB alternative. Developed software is freely available for both commercial and educational purposes.

4. Integration of the method of the moments based solver with ELMER FEM

Recently, ELMER FEM open-source software became commonly accepted standard for finite elements method based scientific calculations. For this reason, it is very useful to create the possibility of integration of MoM solver with ELMER FEM. This is not the trivial task, as ELMER FEM operates on optimisation-based methods of solving of differential equations, whereas for MoM, solving of large scale sets of linear equations is necessary. For this reason following approach was proposed:

a) 2D description the thin layer should be provided accordingly to the .unv standard. This can be created just as a text file, in the case of simple objects or using specialized, open-source software for meshes development, such as SALOME,
b) 2D triangular mesh should by created on the base of .unv file, using ELEM MESH module. This mesh should be generated in a native ELMER FEM format,
c) 2D triangular mesh in a native ELMER FEM format can be loaded to OCTAVE using mesh-import module. During the data import, element clustering should be considered. However, additional information about the element's edges can be

neglected. In the case of MoM this information is not necessary to state border conditions,

d) 2D triangular mesh can be converted to uniform prism-shaped mesh in OCTAVE. Centre of gravity of prism-shaped element (c_{px}, c_{py}) is tested to be located inside each triangle-shaped mesh element, described by three points: (t_{x1},t_{y1}), (t_{x2},t_{y2}), (t_{x3},t_{y3}) using the function presented in the listing 1.

```
function res=IsInTriangle(tx1, ty1, tx2, ty2, tx3, ty3, cpx, cpy)

t0=(tx1.*(ty2-y3)+ty1.*(tx3-tx2)+tx2.*ty3-ty2.*tx3);

t1=(cpx.*(ty3-ty1)+cpy.*(tx1-tx3)-tx1.*ty3+ty1.*tx3)./t0;
t2=(cpx.*(ty2-ty1)+cpy.*(tx1-tx2)-tx1.*ty2+ty1.*tx2)./(-1.*t0);

res=(0<=t1)&(t1<=1)&(0<=t2)&(t2<=1)&(t1+t2<=1);

end
```

Listing.1:*OCTAVE function for testing if the centre of gravity of prism-shaped element (c_{px}, c_{py}) is located inside each triangle-shaped mesh element, described by three points: (t_{x1},t_{y1}), (t_{x2},t_{y2}), (t_{x3},t_{y3}).*

After conversion, the information about material relative permeability as well as layer thickness should be provided,

e) the set of linear equations (2-3) can be determined and solved,
f) the results of simulation may be visualized ad archived accordingly to the user needs.

Presented approach enables determination of magnetization distribution in the thin layers, described as 2D elements in ELMER FEM. Information about the layer thickness and material permeability is provided later, for linear equations solver.

5. Results of the modelling

Figure 2 presents the results of the mesh-conversion as well as the numerical results of the modelling. Triangular cell-based 2D mesh presented in the figure 2a is converted to prism cell based 3D mesh (considering thickness of the layer) presented in the figure 2b. Then, the magnetostatic simulation is carried out, for magnetizing field strength oriented in the x-axis direction. The results of relative value of flux density B_x and B_y distribution is presented in figures 2c and 2d respectively.

6. Conclusion

Presented results confirm, that the method of the moments is suitable andeffective for modelling of magnetostatic magnetization of thin layers. In opposite to the finite elements method, method of the moments requires solving sets of linear equations in spite of solving ill-posed differential equations.

Presents software enables calculations of distribution of magnetization in the thin layers considering the layer's thickness. Moreover, software uses open-source OCTAVE environment, what make it especially useful for advanced sensors development at universities or in small, spin-off enterprises.

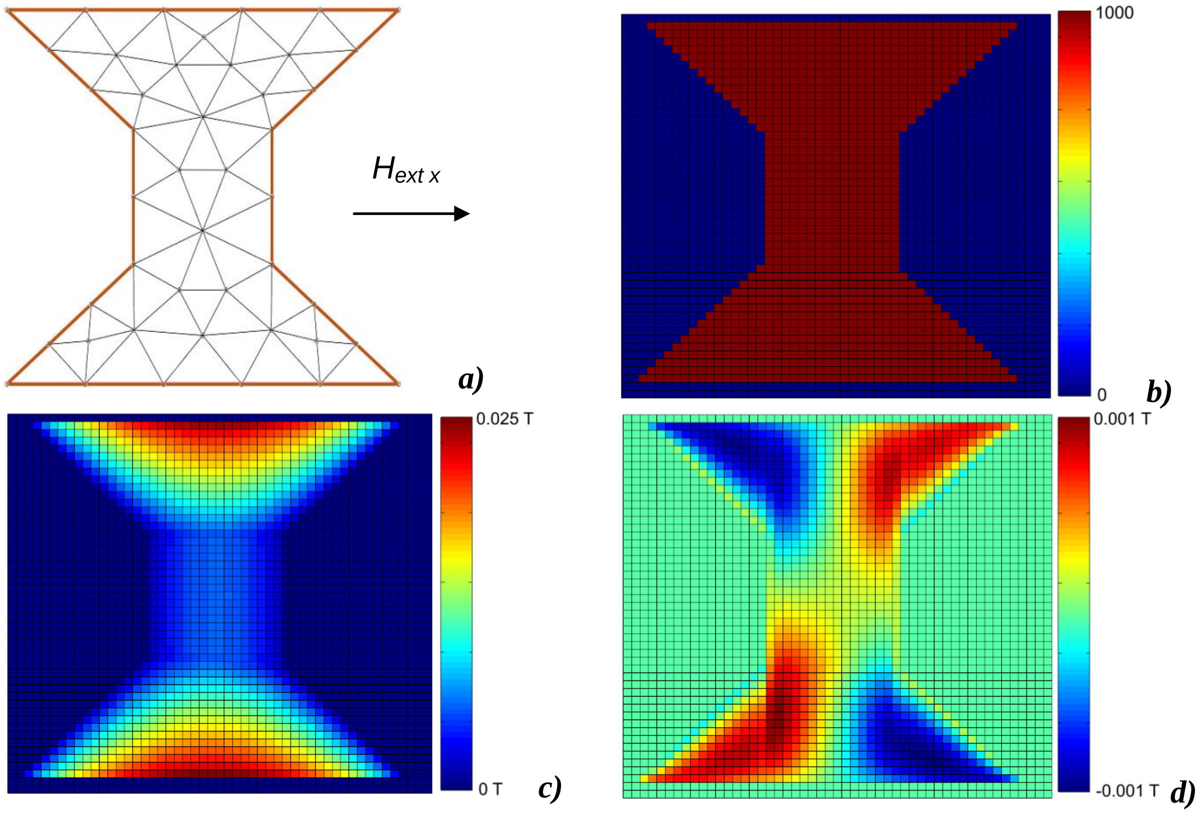

Fig.2:*Analyse of magnetization of candy-shaped element using MoM (layer thickness: 10^{-5}m, layer length in y-axis:10^{-2}m , material relative permeability: 1000,magnetization in y-axis direction): a) triangle-shaped mesh of element, b) mesh converted to prism-shaped elements, c) x-axis flux density B_x distribution, d) y-axis flux density B_y distribution.*

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