MULTI-GROUP TRANSIENT NUMERICAL ANALYSIS OF SLAB NUCLEAR REACTOR

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1. Introduction

Nowadays second recriticality of fast reactor core is discussed based on the steady and transient state neutronic calculations. It is assumed that the molten core is relocated within fixed core boundaries and new core compaction is responsible for second recriticality of nuclear system [1].The design of new reactor concepts has to be investigated for mitigation of such event. Therefore, the neutronic calculation group at Institute of Nuclear and Physical Engineering developed simple 1D deterministic numerical code *TRANSOS* for multigroup transient analysis. The group is actively involved within the ALLEGRO project and the code can be used for academic purposes. Code itself consists of several sub-modules that are able to calculate the steady state by usage of the power method with infinity norm [2][2] .The transient state is calculated by Crank-Nicholson numerical method[3] with the possibility of neutron source incorporation and thermal feedback of microscopic cross sections can be integrated during transient calculation. The whole code is developed in C++ and prepares input files for GNUPLOT plotter. This paper follows the previous work [2] and describes calculation methodology and analysis ofUO_2 system.

2. Steady-state calculation

Brief description of steady-state calculation is introduced in this section. More details can be found in previous work [2].

Code *TRANSOS* solves neutron diffusion equation (1) [4].

$$D_g \Delta\phi_g - \Sigma_{a,g}\phi_g - \sum_h \Sigma_{g\to h}\phi_g + \sum_h \Sigma_{h\to g}\phi_h + \chi_g \sum_h \nu\Sigma_{f,h}\phi_h = 0 \qquad (1)$$

where g represents the group, Σ_a is the absorption macroscopic cross section and D stands for diffusion coefficient. The first term represents the leakage, the second term represents absorptions, third represents disappearances from the group by transfer to another group (h), fourth represents contributions by transfer from another group and last term represents contribution of fission reaction to specified energy group (χ_g is the proportion of neutrons emitted by fission in group g, ν is average number of neutrons produced after fission).

The finite difference method is used for discretization of examined area. Equation (1) can be rewritten for central node of the system (2).The Box Scheme is used for discretization [5].

$$-\tilde{D}_g^{k-1}\phi_g^{k-1} + \left(\tilde{D}_g^{k-1} + \tilde{D}_g^{k} + \Sigma_{rg}^{k}h_k\right)\phi_g^{k} - \tilde{D}_g^{k}\phi_g^{k+1} = \lambda\chi_g^{k}h_k\left(\sum_{g'=1}^{2}\nu\Sigma_{fg'}^{k}\phi_{g'}^{k}\right) + h_k\left(\sum_{\substack{g'=1\\ g'\neq g}}^{2}\Sigma_{s,g'g}^{k}\phi_{g'}^{k}\right) \qquad (2)$$

where k represents spatial node, h_k is length of the k^{th} interval, Σ_f is fission macroscopic cross section, Σ_s stands for scattering cross section between energy groups of neutrons without backscattering. Arbitrary coupling coefficient ($\widetilde{D}$) and removal macroscopic cross section (Σ_f) is shown in (3).

$$\Sigma_{rg}^{k} = \Sigma_{ag}^{k} + \sum_{\substack{g'=1 \\ g' \neq g}}^{2} \Sigma_{s,gg'}^{k} \quad \widetilde{D}_g^k = 2\frac{\dfrac{D_g^k}{h_k}\dfrac{D_g^{k+1}}{h_{k+1}}}{\dfrac{D_g^k}{h_k} + \dfrac{D_g^{k+1}}{h_{k+1}}} \tag{3}$$

The convergence criterion is set for eigenvector with the condition number equal to 1E-04 (more conservative condition number does not increase significantly accuracy in investigated case). When eigenvalues get close to one another then the eigenvectors are not too well defined after iteration process [6]. Therefore, the convergence criterion set for eigenvector is often more conservative than for eigenvalue.

3. Transient-state calculation

The Crank-Nicholson method solves the differential equation in the form of (4).

$$M\frac{\partial^2 U(x,t)}{\partial x^2} = \frac{\partial U(x,t)}{\partial t} + f(x,t) \tag{4}$$

It uses the finite difference representation of diffusion equation and solves the set of simultaneous algebraic equations to get the values of unknowns at the grind point (node). In this method central difference approximation is developed at mid point of time increment [3]. According to difference approximation the (4) can be rewritten for neutron diffusion equation (5).

$$\begin{aligned} &-\frac{\widetilde{D}_g^{k-1}h_t}{2h_k}\phi_g^{k-1,j} + (1+\frac{\widetilde{D}_g^{k-1}h_t}{2h_k} + \frac{\widetilde{D}_g^{k}h_t}{2h_k})\phi_g^{k,j} - \frac{\widetilde{D}_g^{k}h_t}{2h_k}\phi_g^{k+1,j} = \\ &= \frac{\widetilde{D}_g^{k-1}h_t}{2h_k}\phi_g^{k-1,j+1} + (1-\frac{\widetilde{D}_g^{k-1}h_t}{2h_k} - \frac{\widetilde{D}_g^{k}h_t}{2h_k})\phi_g^{k,j+1} + \frac{\widetilde{D}_g^{k}h_t}{2h_k}\phi_g^{k+1,j+1} + \\ &+ \chi_g^k h_t\left(\sum_{g'=1}^{2}\nu\Sigma_{fg'}^{k}\phi_{g'}^{k,j}\right) - \Sigma_{rg}^{k}h_t\phi_g^{k,j} + h_t\left(\sum_{\substack{g'=1 \\ g'\neq g}}^{2}\Sigma_{s,g'g}^{k}\phi_{g'}^{k,j}\right) + S_g h_t \end{aligned} \tag{5}$$

where index j represents step time, h_t is length of the j^{th} time interval and S represents external neutron source.

Integro-differential equation (6)is solved for heat diffusion equation. The produced heat is dependant from previous neutron diffusion equation.

$$-K\frac{\partial^2 T(x,t)}{\partial x^2} + \frac{\partial T(x,t)}{\partial t} = \int_0^t M(x,\tau,T(x,\tau))\cdot d\tau + f(x,t) \tag{6}$$

where K represents material conductivity, and M is function of temperature evolution. The above mentioned problem describes the system where the intensity of heat generation is dependant from already produced heat[7]. In our case it is represented by thermal feedback on macroscopic cross sections in (5). Equation (6) can be similarly rewritten for the heat diffusion equation (7) [8].

$$-\frac{\tilde{\tilde{K}}^{k-1}h_t}{2h_k}T^{k-1,j}+(1+\frac{\tilde{\tilde{K}}^{k-1}h_t}{2h_k}+\frac{\tilde{\tilde{K}}^{k}h_t}{2h_k})T^{k,j}-\frac{\tilde{\tilde{K}}^{k}h_t}{2h_k}T^{k+1,j}=$$
$$=\frac{\tilde{\tilde{K}}^{k-1}h_t}{2h_k}T^{k-1,j+1}+(1-\frac{\tilde{\tilde{K}}^{k-1}h_t}{2h_k}-\frac{\tilde{\tilde{K}}^{k}h_t}{2h_k})T^{k,j+1}+\frac{\tilde{\tilde{K}}^{k}h_t}{2h_k}T^{k+1,j+1}+$$
$$+\left(\sum_{g'=1}^{2}\Sigma_{fg'}^{k}\phi_{g'}^{k,j}\right)E\frac{h_t}{\rho_k c_k}e \qquad (7)$$

$$\tilde{\tilde{K}}^{k}=2\frac{\frac{K^k}{h_k\rho_k c_k}\frac{K^{k+1}}{h_{k+1}\rho_{k+1}c_{k+1}}}{\frac{K^k}{h_k\rho_k c_k}+\frac{K^{k+1}}{h_{k+1}\rho_{k+1}c_{k+1}}}$$

where K represents material conductivity, ρ material density, c material heat capacity, E the average released energy per fission in eV and parameter e is electron charge.

4. Calculational conditions

Zero incoming current boundary condition is used for the neutron diffusion equation (8).

$$\alpha=\frac{J}{\phi}=0.5 \qquad (8)$$

where J is neutron current and ϕ neutron flux.

The absolute value of neutron flux distribution is calculated from proportional neutron flux distribution (2) by setting up initial power generation condition (9).

$$P=\int_V E\cdot\left(\sum_{g'=1}^{2}\Sigma_{fg'}\phi_{g'}\right)\cdot e\cdot dV \qquad (9)$$

The power generation rate is calculated from the burn up of typical VVER fuel assembly (50 MW/kg) and it is approximately 58.74 kW/cm per assembly. Temperature of whole system is set to 296.3 K before transient. Dirichlet boundary condition is also set to 296.3 K for heat diffusion equation.

5. Reactor core under consideration

The investigated geometry represents middle cut of one heterogenic VVER440 fuel assembly placed in the middle (Fig. 1). One edge of investigated geometry is represented by homogenously representation of heterogeneous fuel assembly. Control rod assembly is placed on the opposite edge. This case is introduced due to simulation of transient state during operational conditions.

Geometry is also simplified into one dimension and some lengths are changed for faster convergence of numerical calculation. Length (*d1*) of UO_2 material is 7 mm and is also same for UO_2 + Gd material. Enrichment of UO_2 is 22 % and in gadolinium composition 21.4% with 3.35% of ^{153}Gd.Absorption material B_4C is placed on one edge of core configuration with length (*d5*) 142 mm. The homogenous mixture of area between x – y points is placed on the other edge of nuclear system with length (*d5*).H_2O is placed in the rest of the core configuration with lengths (*d2*) 5 mm, (*d3*) 19 mm and (*d4*) 10 mm. All cross sections are obtained from ENDF/B-VII.1 library for temperatures 296.3 K, 600 K and 1800 K for the discrete energies 0.0253 eV and 2 MeV [9]. Isotropic scattering is assumed and the diffusion coefficient can be calculated according to (10) [10].

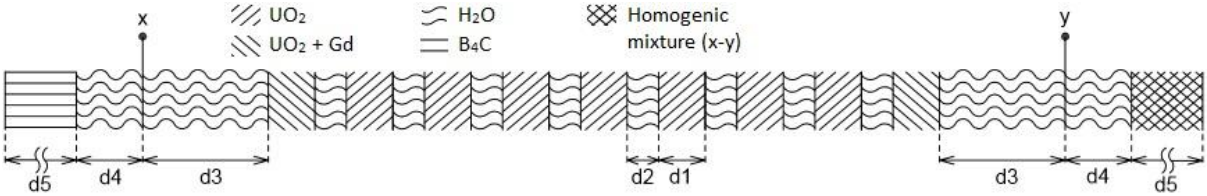


Fig.1:*Core configuration.*

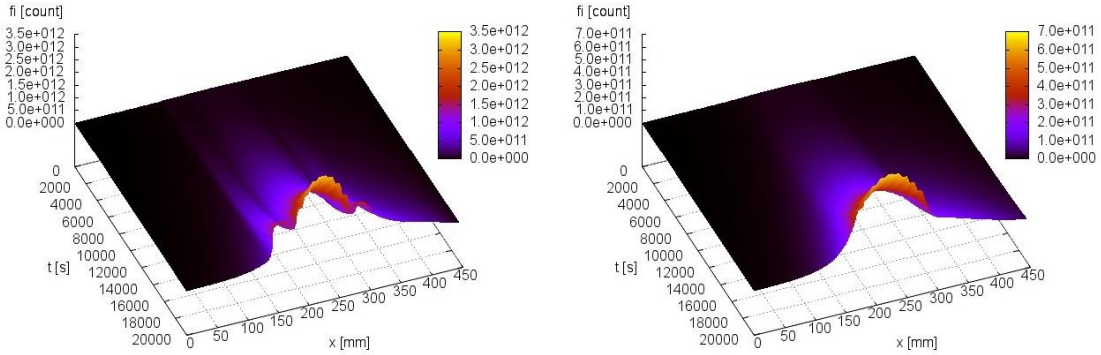
$$D \cong \frac{1}{3\Sigma_t} \tag{10}$$

where Σ_t stands for the total macroscopic cross section.

More details about core configurationcan be found in [2]. It is necessary to note that this is only theoretical approximation and that is mainly because the values of macroscopic scattering cross section were simplified with certain degree of uncertainty.

6. Results

From the steady-state calculation is obtained k_{eff} = 0.982693 for the investigated core configuration. However, in Fig. 2 is possible to see that the neutron flux is increasing during transient, even though the system is subcritical according to calculated multiplication factor from steady-state analysis. It is caused by temperature evolution within the core configuration (Fig. 3).



a) *b)*

Fig.2:*Neutron flux distribution for: a) thermal energy group, b) epithermal energy group.*

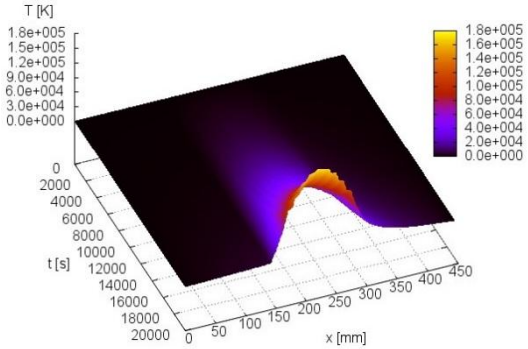


Fig.3: *Temperature distribution of core configuration.*

Applied temperature feedback has influence on the microscopic cross section data of the core configuration. The power is generated in nuclear system by (9). No fluid mechanics is applied and that causes the drastic temperature increase. In addition no thermo-mechanical

physics is applied so the densities of particular materials are independent from temperature evolution. The k_{eff} at the end of the transient state is equal to 1.000466, which is slightly supercritical state.

7. Conclusion

Successful application of multigroup transient numerical code *TRANSOS* was demonstrated. The influence of microscopic cross section temperature feedback caused that the core configuration passed through subcritical state to slightly supercritical state. The maximum values of neutron flux and temperature are located in heterogeneous region. However, the diffusion theory is more appropriate for homogenous problems and calculation error increases with greater heterogeneity. In the future the contribution of ^{238}U thermal feedback with thermo-hydraulic properties of the system has to be investigated.

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