STABILITY ANALYSIS OF NUCLEAR REACTOR WITH CHANGE IN EIGENVALUE – CYLINDRICAL GEOMETRY

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1. Introduction

The paper is focused on the method of determination of nuclear reactor stability. This work arose from collaboration of B&J NUCLEAR ltd. with Korean Atomic Energy Research Center within the project "Static stability analysis methodology development". Method is based on the paper "Stability of Nuclear Reactors with Changes in Eigenvalue" [1]. Reactor stability is evaluated by change in eigenvalue – change in the geometry or the material buckling. Reactor with prompt feedback due to the Doppler effect on resonance absorption is considered. Using the perturbation method, equilibrium state is obtained and its temporal response to initial deviation is examined. At the original study [1], a slab bare homogeneous reactor was used. In this paper, the method was derived for cylindrical geometry.

2. Equilibrium state

In case of a homogeneous slab reactor, the one-group diffusion equation with spatial domain *D* can be written as:

$$\frac{1}{v}\frac{\partial\psi(r,t)}{\partial t} = D\Delta\psi + [k(T,\psi) - 1]\Sigma_a\psi, \tag{1}$$

where ψ stands for neutron flux, k is multiplication factor, D represents diffusion coefficient, Σ_a is macroscopic absorption cross section and v stands for neutron velocity.Functional dependence of neutron multiplication factor k on the state variable temperature T and neutron flux ψ is given by:

$$k(\psi, T_c) = k_0 + a_1 ln\frac{\psi(r,t) + b(r)}{\psi_0(r) + b(r)}. \tag{2}$$

Character a_1 contains Doppler constant [2], which is supposed to be positive or negative. The subscript *0* represents initial state of equilibrium. Character $b(r)$ is space-dependent constant containing Doppler constant and the mean time of heat transfer to coolant. Substituting Eq. (2) into Eq. (1) and using nondimensionalised variables $x = (\Sigma_a/D)^{1/2}r$ and $\tau = v\Sigma_a t$ leads to:

$$\frac{\partial\psi(x,\tau)}{\partial\tau} = \Delta\psi + \left[\lambda + a_1 ln\frac{\psi(x,\tau)+b(x)}{\psi_0(x)+b(x)}\right]\psi(x,\tau) \qquad \text{in } D,$$
$$\psi(x,\tau) = 0 \quad \text{on} \quad \partial D, \tag{3}$$

where

$$\lambda = (k_0 - 1). \tag{4}$$

The equilibrium states are the solutions of the associated boundary value problem defined by

$$\Delta\psi + \left[\lambda + a_1 ln\frac{\psi(x)+b(x)}{\psi_0(x)+b(x)}\right]\psi(x) = 0 \qquad \text{in } D$$
$$\psi(x) = 0 \quad \text{on} \quad \partial D. \tag{5}$$

Eq. (5) has the standard eigenfunctions and eigenvalues,

$$\psi_{0n}(x) = A_n^{(0)}\phi_n(x). \tag{6}$$

In the case of a slab reactor, eigenfunctions and eigenvalues are given by

$$\phi_n(x) = \cos(\frac{n\pi}{2}x)$$
$$\lambda_n(0) = (\frac{n\pi}{2})^2 \qquad n \text{ odd}. \tag{7}$$

Before the perturbation is introduced, the reactor is supposed to be at the equilibrium state, described by the fundamental mode of Eq. (6) with eigenvalue $(\frac{\pi}{2})^2$[3].
Eq. (5) is solved using the perturbation method. Eigenvalue $\boldsymbol{\lambda_n(\varepsilon)}$ and equilibrium state can be found in the following form:

$$\lambda_n(\varepsilon) = \lambda_n(0) - \frac{\varepsilon^2}{2}a_1\int_{-1}^{1}\frac{A_n^{(0)}[\phi(x) + 2\mathrm{b(x)}]}{[A_n^{(0)}\phi(x) + b(x)]^2}\phi(x)^3 dx + 0(a_1^2) \tag{8}$$

and

$$\psi_n(x,\varepsilon) = A_n^{(0)}\left[(1+\varepsilon)\phi(x) + \varepsilon a_1\sum_{\substack{j=1\\ j\neq n}}^{\infty}\frac{\phi(x)}{\lambda_j(0)-\lambda_n(0)}\int_{-1}^{1}\frac{A_n^{(0)}\phi^2(x')\phi(x')}{A_n^{(0)}\phi(x')+b(x')}dx' + \sum_{r=2}^{\infty}\frac{\varepsilon^r}{r!}\int_{-1}^{1}S^{(r)I}(x')G_0(x,x')dx' + 0(\varepsilon a_1^2)\right], \tag{9}$$

where $\boldsymbol{\varepsilon}$ stands for perturbation, r is an order of derivation with the respect to the ε, $G_0(x,x')$ is the Green's function and $A_n^{(0)}$ is an integration constant which can be obtained from power condition.It should be noted that Eq. (8) represents solution to the first order of Doppler feedback and to the to the higher orders of perturbation.
Since the cylindrical geometry is more realistic, to know appropriate formulas in cylindrical coordinates seems to be meaningful. In the cylindrical reactor, the flux depends on the radius *r*, height *z*, and polar angle ϑ. To keep our case as simple as possible, the reactor consisting of a homogeneous material mixture is assumed, therefore the angular dependence of the flux can be neglected. In order to find standard eigenfunctions and eigenvalues for the cylindrical geometry, diffusion equation is written in the form:

$$\frac{\partial^2\phi(r,z)}{\partial r^2}+\frac{1}{r}\frac{\partial\phi(r,z)}{\partial r}+\frac{\partial^2\phi(r,z)}{\partial z^2}+B_g^2\phi(r,z)=0. \quad (10)$$

Separation-of-variables technique is used to solve Eq. (10):

$$\phi(r,z)=\chi(r)Z(z) \quad (11)$$

In the case of a cylindrical reactor, eigenfunctions and eigenvalues are given by

$$\phi_n(z,r)=\cos(\frac{n\pi}{H}z)J_0(\nu_n r)$$
$$\lambda_n(0)=(\frac{n\pi}{H})^2+(\nu_n)^2 \qquad n\,\text{odd}. \quad (12)$$

Term $J_0(\nu_n r)$ is reffered as Bessel function of zeroth order. [4]. Whereas the method of diffusion equation solution is the same as for slab geometry, formulas for the flux can be written directly. For the higher-order term of perturbation, the solution is given by the following equation:

$$\lambda_n(\varepsilon)=\lambda_n(0)-\frac{\varepsilon^2}{2}a_1x\int_r\int_z r\frac{A_n^{(0)}\left[A_n^{(0)}\phi_n(r,z)+2b(r,z)\right]}{\left[A_n^{(0)}\phi_n(r,z)+b(r,z)\right]^2}x\phi_n^3(r,z)drdz+0(a_1^2) \quad (13)$$

and

$$\psi_n(r,z,\varepsilon)=A_n^{(0)}\left[\phi_n(r,z)+\varepsilon a_1\sum_{\substack{j=1\\j\neq n}}^{\infty}\frac{\phi_j(r,z)}{\lambda_j(0)-\lambda_n(0)}\int_r\int_z r\frac{A_n^{(0)}\phi_n^2(r,z)\phi_j(r,z)}{A_n^{(0)}\phi_n(r,z)+b(r,z)}drdz+\sum_{l=2}^{\infty}\frac{\varepsilon^l}{l!}\int_r\int_z rS^{(r)l}(r,z)G_0(r,z)drdz+0(\varepsilon a_1^2)\right]. \quad (14)$$

It could be seen that formulas for the flux are almost the same as for the slab geometry, except $\phi_n(r,z)$ is now given by Eq. (12) and Jacobi operator r was applied to transform integrals from Cartesian to Cylindrical coordinates. It should be noted that l stands for an order of derivation with respect to ε.[4]

3. Stability analyses

Stability of the equilibrium state is examined by considering the temporal response of the initial deviation η from the equilibrium value $\psi_n(r,z,\lambda)$. Accordingly, solution should be found in the following form

$$\psi_n(r,z,\tau,\varepsilon,\eta)=\psi_n(r,z,\varepsilon)+\eta\psi_\eta(r,z,\tau,\varepsilon), \quad (15)$$

where$\psi_n(r,z,\varepsilon)$ are defined by Eq. (14). To find ψ_η, Eq. (3) is differentiated once with the respect to the η. Next assumption is made

$$\psi_\eta(r,z,\varepsilon)=u(r,z,\varepsilon)exp[\beta(\varepsilon)\tau]. \quad (16)$$

Thestability of the fundamental mode of the flux can be examined up to the second order in ε, by the following condition

$$\beta(\varepsilon) = \varepsilon\dot{\beta}(0) + \frac{\varepsilon^2}{2}\ddot{\beta}(0) < 0, \qquad (17)$$

leading to the formula

$$\varepsilon < 2\int_r \int_z r\frac{A_n^{(0)}[\psi_{0n}(r,z)+2b(r,z)]}{[\psi_{0n}(r,z)+b(r,z)]^2}\phi^3(r,z)\left\{\int_r \int_z r\frac{A_n^{(0)}[\psi_{0n}(r,z)+2b(r,z)]}{[\psi_{0n}(r,z)+b(r,z)]^2}\phi^3(r),z)drdz + \int_r \int_z r\frac{[A_n^{(0)}]^2[\psi_{0n}(r,z)+3b(r,z)]}{[\psi_{0n}(r,z)+b(r,z)]^3}\phi^4(r,z)drdz\right\}^{-1}. \qquad (18)$$

The results are summarized in Table 1. Based on these results, the following physical interpretation of the results can be given. In the case of a cylindrical reactor with positive feedback (a_1>0) and with λ smaller than the critical buckling of the reactor, two fundamental equilibrium states exist. Given an initial disturbance η, the larger state ($\varepsilon > 0$) is unstable, while the smaller state ($\varepsilon < 0$) is stable. Nevertheless, the state with $\varepsilon > 0$ with $\lambda > (\frac{\pi}{H})^2 + (\frac{2.405}{R})^2$is rejected as physically impossible. If the feedback is negative, then two equilibrium states exist for $\lambda > (\frac{\pi}{H})^2 + (\frac{2.405}{R})^2$.

The state with ($\varepsilon > 0$), $\lambda > (\frac{\pi}{H})^2 + (\frac{2.405}{R})^2$ and a_1<0 is of our special interest. It could be seen, that even in case of a negative feedback, there is a limit to the increase of the reactor eigenvalue, beyond which instability may result.

Tab. 1.*Stability of Reactors with Changes in Eigenvalue.*

Case	Eigenvalue	Stability condition
$\boldsymbol{\varepsilon} > 0, \boldsymbol{a_1} < 0$	$\boldsymbol{\lambda} > \boldsymbol{\lambda_1(0)}$	$\boldsymbol{\varepsilon} < 2\int_{-1}^{1}\frac{(\boldsymbol{\psi_{01}}+\boldsymbol{2b})}{(\boldsymbol{\psi_{01}}+\boldsymbol{b})^2}\boldsymbol{\phi}^3\,\boldsymbol{dx}\{\int_{-1}^{1}[\frac{(\boldsymbol{\psi_{01}}+\boldsymbol{2b})\boldsymbol{\phi}^3}{(\boldsymbol{\psi_{01}}+\boldsymbol{b})^2} + \frac{\boldsymbol{A_1^{(0)}}(\boldsymbol{\psi_{01}}+\boldsymbol{3b})\boldsymbol{\phi}^4}{(\boldsymbol{\psi_{01}}+\boldsymbol{b})^3}]\boldsymbol{dx}\}^{-1}$
$\boldsymbol{\varepsilon} < 0, \boldsymbol{a_1} < 0$	$\boldsymbol{\lambda} > \boldsymbol{\lambda_1(0)}$	Unstable but physically impossible
$\boldsymbol{\varepsilon} > 0, \boldsymbol{a_1} > 0$	$\boldsymbol{\lambda} < \boldsymbol{\lambda_1(0)}$	Unstable but physically impossible
$\boldsymbol{\varepsilon} < 0, \boldsymbol{a_1} > 0$	$\boldsymbol{\lambda} < \boldsymbol{\lambda_1(0)}$	Stable

4. Practical use of derived methodology

To put the results summarized in Table 1 into practice, the following steps should be done:

- It is necessary to identify temperature dependence of multiplication factor. The effective multiplication factors k_1, k_2 and k_3 are calculated for three different temperatures $T_1, T_2$and T_3.
- The user should calculate the Doppler coefficient K of the examined system.
- To obtain constants $b(r)$ and K', the coolant temperature $T_c(r)$ and the mean time of heat transfer γ is required.
- Non-dimensionalised variables $x$and τ, defined as $x = (\Sigma_a/D)^{\frac{1}{2}}r$ and $\tau = v\Sigma_a t$, are used in Eq. (3). To obtain flux values, the macroscopic absorption cross section Σ_a, the diffusion coefficient D and the effective neutron velocity v should be determined.

- The flux values ψ_{01} are given by Eq. (6), where $\phi_1(x)$ can be calculated from Eq. (7) using the above mentioned formulas for x, and constant $A_1^{(0)}$ can be determined from the power condition
- Once all mentioned parameters and constants are known, the value of perturbed λ can be calculated by Eq. (8). Accordingly, the reactor stability can be examined using parameters presented in Table 1. Based on the known parameters and constants, the limit value of perturbation ε, given by Eq. (18), can be calculated as well.

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