MODELLING OF ELASTO-STATICS OF POWER LINES BY NEW COMPOSITE BEAM FINITE ELEMENT

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1. Introduction

The power line is from mechanical point of view a 3D system. It can be loaded in longitudinal, horizontal and vertical direction. Also the torsional loading is possible as well. But in the technical calculations it is mostly simplified to the one dimensional system. In the literature, e. g [1], an analytical method is presented for the power line elasto-static analysis in its vertical plane. The analytical methods are not much effective for the general spatial analysis. The more effective are the numerical methods, over all the finite element method. For the simple elasto-static analysis the geometrically nonlinear link finite element can be used that is able to analyze the tensional forces and stresses, and the elongation of the line, e.g. [2]. For dynamic analysis, the beam finite element is preferable [3]. The heterogeneous cross-section are of several construction, e.g. as shown in Fig.1 [4]. Because the material of the power line is inhomogeneous, the homogenization of material properties is needed. Therefore, the axial, flexural and torsional stiffness must be stated. For the mechanical analysis also the solid finite elements are available, but modeling of the complicated geometry and heterogeneity is a very demanding procedure.

In this paper, the results of elasto-static analysis of the single and bundle AlFe power lines are presented.

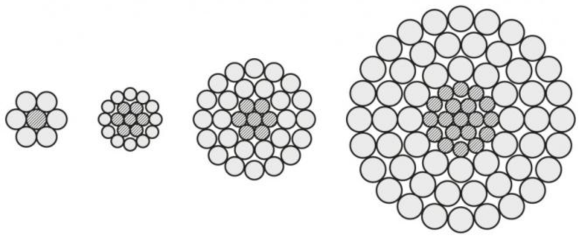

Fig. 1. *Construction of AlFe power line cross-section* [4].

For modeling and simulation of the problem a new 3D composite beam finite element is used, which was developed at our institute [3]. The second order beam theory has been used for the finite element stiffness matrix formulation. The effective tensional, flexural and torsional stiffness of chosen power lines are considered [3]. The results are calculated, evaluated and compared with the ones obtained by the standard finite element software [2].

2. Composite finite beam element equations

Let us consider a 3D straight finite beam element (Timoshenko beam theory and Saint-Venant torsion theory) of doubly symmetric cross-section – Fig. 2. The nodal degrees of freedom at node i are: the displacements u_i, v_i, w_i in the local axis direction x, y, z, and the cross-sectional area rotations - $\varphi_{x,i}, \varphi_{y,i}, \varphi_{z,i}$. The degrees of freedom at the node j are denoted in a similar manner. The internal forces at node i are: the axial force N_i, the transversal forces $R_{y,i}$ and $R_{z,i}$, the bending moments $M_{y,i}$ and $M_{z,i}$, and the torsion moment $M_{x,i}$. The first derivative with respect to x of the relevant variable is denoted with an apostrophe " ′ ".

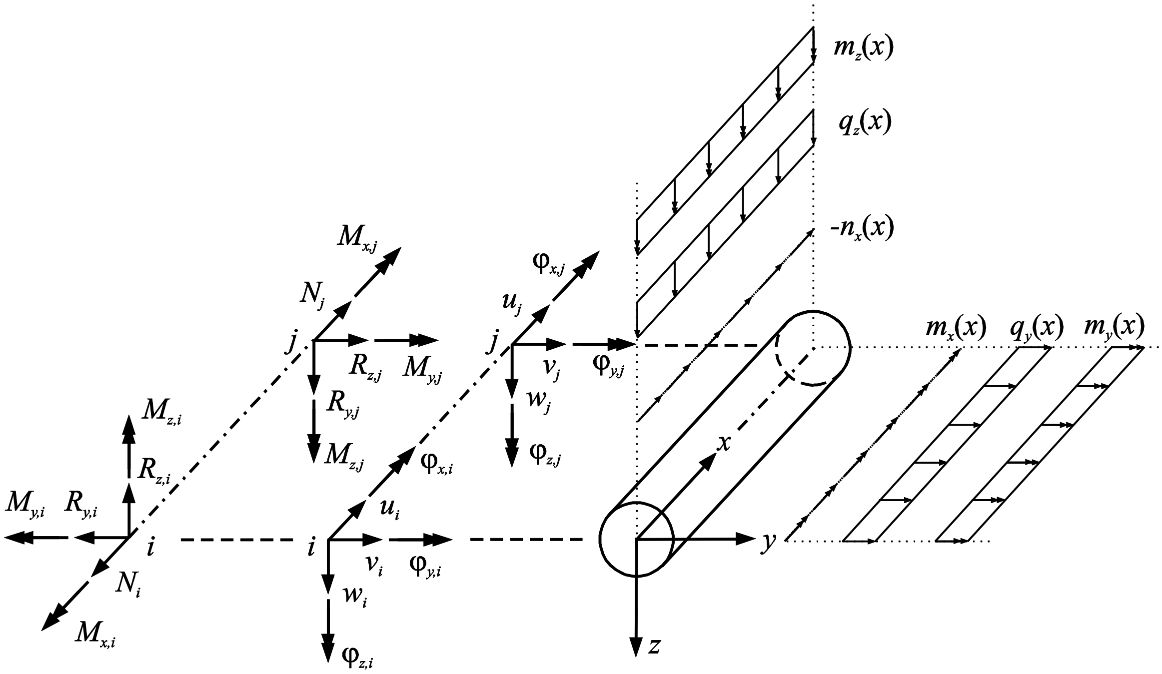

Fig. 2: *The local internal variables and loads.*

Furthermore, $n_x = n_x(x)$ is the axial force distribution, $q_z = q_z(x)$ and $q_y = q_y(x)$ are the transversal and lateral force distributions, $m_x = m_x(x)$, $m_y = m_y(x)$ and $m_z = m_z(x)$ are the distributed moments, A is the cross-sectional area, I_y and I_z are the second area moments, $I_p = I_y + I_z$ is the area polar moment. The effective homogenized and longitudinally varying stiffness reads: $EA = E_L^{NH}(x)A$ is the axial stiffness ($E_L^{NH}(x) \equiv E_L^{NH}$ is the effective elasticity modulus for axial loading), $EI_y = E_L^{M_yH}(x)I_y$ is the flexural stiffness about the y-axis ($E_L^{M_yH}(x) \equiv E_L^{M_yH}$ is the effective elasticity modulus for bending about axis y), $EI_z = E_L^{M_zH}(x)I_z$ is the flexural stiffness about the axis z, ($E_L^{M_zH}(x) \equiv E_L^{M_zH}$ is the effective elasticity modulus for bending about axis z), $G\bar{A}_y = G_{Ly}^H(x)k_y^{sm}A$ is the reduced shear stiffness in y – direction ($G_{Ly}^H(x) \equiv G_{Ly}^H$ is the effective shear modulus and k_y^{sm} is the average shear correction factor in y – direction [5]), $G\bar{A}_z = G_{Lz}^H(x)k_z^{sm}A$ is the reduced shear stiffness in z – direction ($G_{Lz}^H(x) \equiv G_{Lz}^H$ is the effective shear modulus and k_z^{sm} is the average shear correction factor in z – direction [5]), $G_L^{M_xH}(x)I_T$ is the effective torsional stiffness ($G_L^{M_xH}(x) = G_L^{M_xH}$ is the torsional elasticity modulus and I_T is the torsion constant). Detailed derivation of the effective material properties is presented in [3] and [4]. Establishing of the local composite beam finite element equations is presented in [6]:

$$
\begin{bmatrix} N_i \\ R_{y,i} \\ R_{z,i} \\ M_{x,i} \\ M_{y,i} \\ M_{z,i} \\ N_j \\ R_{y,j} \\ R_{z,j} \\ M_{x,j} \\ M_{y,j} \\ M_{z,j} \end{bmatrix} =
\begin{bmatrix}
K_{1,1} & 0 & 0 & 0 & 0 & 0 & K_{1,7} & 0 & 0 & 0 & 0 & 0 \\
 & K_{2,2} & 0 & 0 & 0 & K_{2,6} & 0 & K_{2,8} & 0 & 0 & 0 & K_{2,12} \\
 & & K_{3,3} & 0 & K_{3,5} & 0 & 0 & 0 & K_{3,9} & 0 & K_{3,11} & 0 \\
 & & & K_{4,4} & 0 & 0 & 0 & 0 & 0 & K_{4,10} & 0 & 0 \\
S & & & & K_{5,5} & 0 & 0 & 0 & K_{5,9} & 0 & K_{5,11} & 0 \\
 & Y & & & & K_{6,6} & 0 & K_{6,8} & 0 & 0 & 0 & K_{6,12} \\
 & & M & & & & K_{7,7} & 0 & 0 & 0 & 0 & 0 \\
 & & & M & & & & K_{8,8} & 0 & 0 & 0 & K_{8,12} \\
 & & & & E & & & & K_{9,9} & 0 & K_{9,11} & 0 \\
 & & & & & T & & & & K_{10,10} & 0 & 0 \\
 & & & & & & R & & & & K_{11,11} & 0 \\
 & & & & & & & Y & & & & K_{12,12}
\end{bmatrix} \cdot
\begin{bmatrix} u_i \\ v_i \\ w_i \\ \varphi_{x,i} \\ \varphi_{y,i} \\ \varphi_{z,i} \\ u_j \\ v_j \\ w_j \\ \varphi_{x,j} \\ \varphi_{y,j} \\ \varphi_{z,j} \end{bmatrix} \tag{1}
$$

In (1), the terms $K_{i,j}$ contain the linear and the linearized geometric non-linear stiffness terms – containing the axial force effect on the flexural beam stiffness. Shear correction is accounted as well. The global stiffness matrix of the beam structures can be done by classical methods. Establishing of the local and global stiffness matrices as well as the whole solution procedure were coded by the software MATHEMATICA [7].

3. Numerical experiments

The symmetric power line marked as 450AlFe6 (3+9 steel and 11+17 aluminum wires), which is loaded in its initial state by the self-weight in *y*-direction, has been considered. Span of the power line is L = 300 m, the maximal deflection is $y_{\max} = 3{,}966\,\mathrm{m}$ and the average internal axial force is N^{II} = 49.566 kN. The power line is subsequently loaded at its midpoint by forces $F_y = -100\,\mathrm{N}$ and $F_z = 100$ N. The diameter of the aluminum wires is d_{Al} = 4,5 mm and the diameter of the steel wires is d_{Fe} = 2,8 mm. The effective cross-sections of the power line parts are: A_{Fe} = 73,89 mm^2, A_{Al} = 445,32 mm^2 and the effective cross-sectional area of the power line is A = 519,21 mm^2.

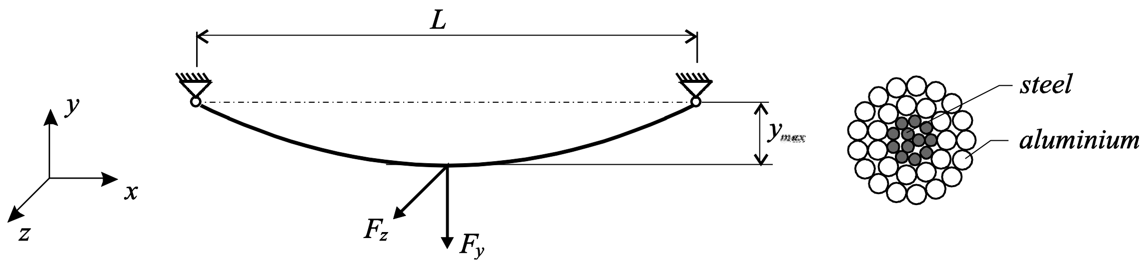

Fig. 3: *AlFe power line in initial state consequently loaded by concentrated forces.*

Material properties of the components are constant and their values are: aluminum – the elasticity modulus $E_{Al} = 70\,\mathrm{GPa}$, the Poisson's ratio $\nu_{Al} = 0{,}32$, the mass density $\rho_{Al} = 2700\,\mathrm{kgm^{-3}}$; steel – the elasticity modulus $E_{Fe} = 210\,\mathrm{GPa}$, the Poisson's ratio $\nu_{Fe} = 0{,}28$, the mass density $\rho_{Fe} = 7850\,\mathrm{kgm^{-3}}$.

For the elasto-static analysis of the single power lines the following effective material properties have been used [3]:

- the effective elasticity modulus for axial loading - $E_L^{NH} = 89923{,}76\,\mathrm{MPa}$
- the flexural stiffness - $E_L^{M_yH} = E_L^{M_zH} = 97146{,}55\,\mathrm{MPa}$

- the effective shear modulus - $G_{Ly}^{H} = G_{Lz}^{H} = 34586{,}06\,\text{MPa}$
- the torsional elasticity modulus - $G_{L}^{M_x H}(x) = 28090{,}28\,\text{MPa}$

The following calculations were done with our 3D FGM beam finite element (NFE). The same problem has been solved using a commercial FEM program ANSYS with 300 of beam finite elements (BEAM188). The global displacements *v* [m] in (*y* – direction) and *w* [m] in (*z* – direction) at distances *x* of 50, 100 and 150 m from the left line end, calculated with ANSYS as well as with the NFE are presented in Table 1. The average relative difference Δ [%] between displacements calculated by our NFE-method and the ANSYS solution has been evaluated, as well.

Tab. 1. *Spatial displacements at the selected points* (DSP) *of the AlFe power line.*

DSP		NFE	ANSYS	Δ [%]
50	v_{50}	-2,2229	-2,2255	0,12
	w_{50}	0,0499	0,0497	0,40
100	v_{100}	-3,5776	-3,5802	0,07
	w_{100}	0,0999	0,0994	0,50
150	v_{150}	-4.0619	-4,0627	0,02
	v_{150}	0,1492	0,1491	0,07

The comparison of total displacement of the AlFe power line calculated by our new approach and commercial FEM program ANSYS is shown in Figure 4.

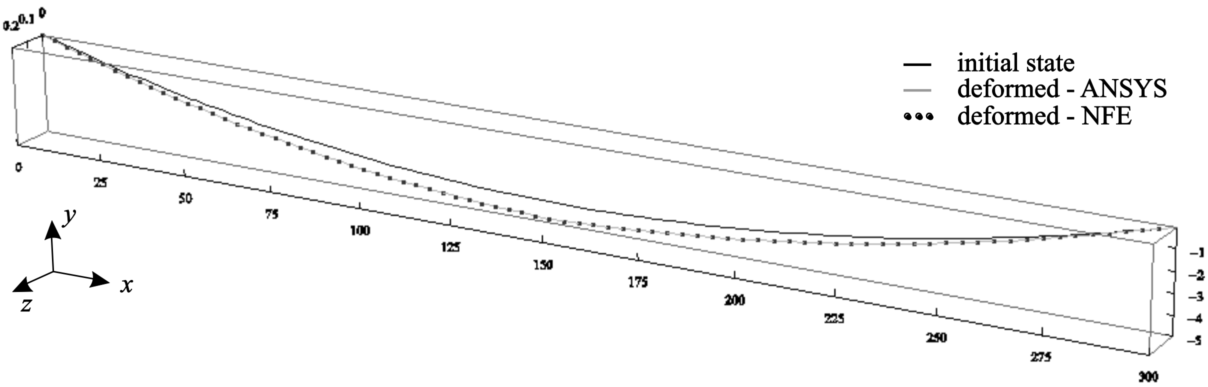

Fig. 4: *Total displacement of the power line.*

The local internal forces and moments in the power line can be calculated from the local displacements at the finite elements nodal points. According to the given loads and the low power line stiffness for bending, the relevant internal force is the axial tensional force $N(x)$. Its maximal value is at the left and right line end: $N_{\max} = 50{,}324$ kN. The average normal stress is $\sigma(x) = N(x)/A$ and the normal strain $\varepsilon(x) = \sigma(x)/E_L^{NH}$ at the *x*-position. Because of different elasticity modulus of the aluminum and the steel a different normal stress in the material components will be arise. Position of the critical cross section (were the maximal normal force arises) is at the left and right end of the power line. In our case, the maximal stress for the alumina is $\sigma_{\max Al} = [\varepsilon(x)]_{x=0} E_{Al} = 75{,}51$ MPa and for steel $\sigma_{\max Fe} = [\varepsilon(x)]_{x=0} E_{Fe} = 226{,}53$ MPa.

4. Conclusions

New composite beam finite element was used for the elasto-static analysis of the power line. Homogenization of the heterogeneous material properties was made by the reference volume method (RVE). For comparison of the effectiveness and accuracy of the new finite element the same problem was solved by the commercial FEM software ANSYS. A very good agreement of both results has been obtained. The most advantage of the new beam finite element is that the stiffness matrix contains the effect of axial force, and all the stiffness (tensional, flexural and torsional) are calculated by a consistent manner.

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