HOMOGENIZATION OF MATERIAL PROPERTIES OF ALFE POWER LINES

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Received 07 May 2015; accepted 14 May 2015

1. Introduction

Power lines are made from wires of various metals, mainly in order to increase the strength of the cable or to decrease nominal weight of the power line.Different combinations of materials are used in practice, mainly from aluminium with steel core (AlFe).

To model the real AlFe power line with its real construction is complicated due to its complicated geometry and heterogeneity. So the simplified models are used. That is the reason why homogenized material properties have to be calculated. The heterogeneous cross-section are of several construction, e.g. as shown in Fig.1. Because the material of the power line is inhomogeneous, the homogenization of material properties is needed.

In the praxis (e.g. [1]) the effective material properties are calculated by means of the mixture rules. The effective elasticity modulus for tension is calculated, which is then used for the elastostatic and modal analysis of the power lines. It provided only the planar deformation of the line. In [2], the calculation of the effective elasticity modulus for the bending is presented. For the standard 3D beam finite element [3], the elasticity modulus for shear and torsion is calculated from the elasticity modulus for tension and the Poisson ration. These simplifications can decrease the solution results accuracy.

In the proposed contribution, the homogenization of material properties of the AlFe power lineis presented, which is then used for calculation of the of the effective electric, thermal and elastic material properties for the selected power line cross-section.

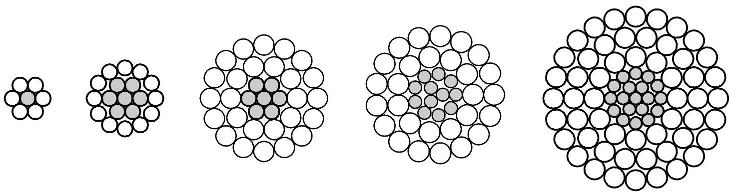

Fig. 1. *Construction of AlFe power line cross-section*.

2. Homogenization theory

One important goal of mechanics of heterogeneous materials is to derive their effectiveproperties from the knowledge of the constitutive laws and complex micro-structural behavior of their components.

The methods based on the homogenization theory (e.g. the mixture rules [4]; self-consistentmethods [5]) have been designed and successfully applied to determine theeffective material properties of heterogeneous materials from the corresponding materialbehavior of the constituents (and of the interfaces between them) and from the geometricalarrangement of the phases. In this context, the microstructure of the material underconsideration is basically taken into account by the Representative Volume Element (RVE).

Modeling of composite structures is the area of interest at our institute. The mixtures rules were extended [6] that are very often used in homogenization of composites properties. There, two different methods for modeling of Functionally Graded Material (FGM) beam with spatial variation of material properties were derived – the multilayered method – MLM (Fig. 2) and later the direct integration method [7]. These homogenization techniques can also be used for homogenization of the AlFe cables.

From the assumption that the corresponding property (electric conductance, the thermal conductance, the thermal expansion, the stiffness) of the real beam is equal to the analogical property of the homogenized beam the homogenized material properties can be calculated.

In case of the power line, the material properties vary layer-wise in the radial direction. The longitudinal variation is not assumed. The MLM has been used in the following consideration.

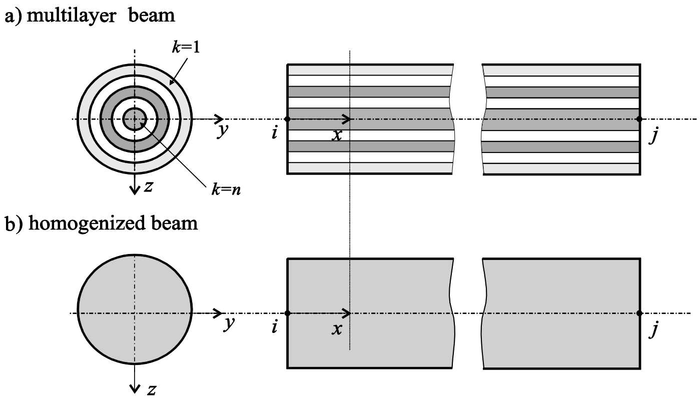

Fig. 2. *Homogenization - multilayered method*.

The effective electric conductance γ_L^H can be calculated by:

$$\gamma_L^H = \frac{\sum_{k=1}^{n} \gamma_k A_k}{A} \tag{1}$$

The effective thermal conductance λ_L^H is:

$$\lambda_L^H = \frac{\sum_{k=1}^{n} \lambda_k A_k}{A} \tag{2}$$

The effective thermal expansion coefficient α_{TL}^H is:

$$\alpha_{TL}^H = \frac{\sum_{k=1}^{n} \alpha_{Tk} E_k A_k}{\sum_{k=1}^{n} E_k A_k} \tag{3}$$

Here, $A = \sum_{k=1}^{n} A_k$ is the cross-sectional area of the whole real cross-section. E_k is the elasticity modulus of the relevant layer, and n is number of the layer.

Since the Young’s modulus multiplied by the cross-sectional area defines the axialstiffness and multiplied by the moment of inertia of cross-section area defines thebending stiffness, we

have to distinguish homogenized effective Young’s modulus foraxial loading E_L^{NH} and homogenized effective Young’s modulus for bending $E_L^{M_y H}$ and $E_L^{M_z H}$.
The effective elasticity modulus for axial loading is:

$$E_L^{NH} = \frac{\sum_{k=1}^{n} E_k A_k}{A} \tag{4}$$

The elasticity modulus for lateral and transversal bending are:

$$E_L^{M_y H} = \frac{\sum_{k=1}^{n} E_k I_{yk}}{I_y}, \qquad E_L^{M_z H} = \frac{\sum_{k=1}^{n} E_k I_{zk}}{I_z} \tag{5}$$

where $I_y = \sum_{k=1}^{n} I_{yk}$ and $I_z = \sum_{k=1}^{n} I_{zk}$ are the quadratic moment of inertia according the axis *y*and according the axis *z*. In many cases holds that $E_L^{M_y H} = E_L^{M_z H}$.
The effective elasticity modulus for lateral and transversal shear

$$G_{Ly}^{H} = \frac{\sum_{k=1}^{n} k_{y,k}^{sm} G_k A_k}{k_y^{sm} A}, \qquad G_{Lz}^{H} = \frac{\sum_{k=1}^{n} k_{z,k}^{sm} G_k A_k}{k_z^{sm} A} \tag{6}$$

where $G_k = E_k / 2(1+\nu_k)$ is the shear modulus of the k^{th}layerand ν_k is its Poisson ratio. Again, $k_{y,k}^{sm}$ and k_y^{sm} is the shear correction factor for the k^{th}layer and the whole cross-section, respectively. These constants have to be calculated by special method [8].
The effective elasticity modulus for torsion is:

$$G_L^{M_x H}(x) = \frac{\sum_{k=1}^{n} G_k I_{pi}}{I_p} \tag{7}$$

where $I_p = I_y + I_z$ is the cross-sectional area polar moment of inertia.
The effective mass density for axial beam vibration is:

$$\rho_L^{NH} = \frac{\sum_{k=1}^{n} \rho_k A_k}{A} \tag{8}$$

The effective mass density for torsional vibration is:

$$\rho_L^{M_x H} = \frac{\sum_{k=1}^{n} \rho_k I_{pk}}{I_p} \tag{9}$$

There, ρ_k is the mass density of the k^{th}layer.

3. Homogenization of the AlFe power line

The power line marked as 450AlFe6 (consist of 3+9 steel and 11+17 aluminum wires)has been considered (as shown in Figure 3), The diameter of the aluminum wires is d_{Al}=4,5 mm and the diameter of the steel wires is d_{Fe} = 2,8 mm.

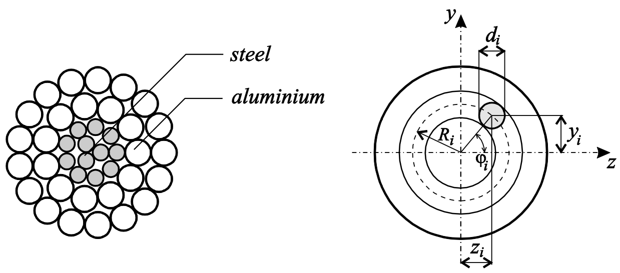

Fig.3: *Power line cross-section.*

Material properties of the components are constant and their values are in Table 1:

Tab. 1. *Material properties of the AlFe power line components.*

Material properties	aluminum	steel
electric conductivity γ [S/m]	$3{,}55\times10^{7}$	1×10^{7}
thermal conductivity λ [$Wm^{-1}K^{-1}$]	237	80,4
elasticity modulus E [GPa]	70	210
Poisson's ratio ν [-]	0,32	0,28
thermal expansion coefficient α_T [K^{-1}]	$23{,}1\times10^{-6}$	$11{,}8\times10^{-6}$
mass density ρ [kgm^{-3}]	2700	7850

The principle of calculation of the quadratic moment of inertia for cable is shown in Fig. 3. Here, R_k is pitch circle, d_k is wire diameter, φ_k is the angle of circumferential position of the wire, z_k and y_k are the distances of the wire from the center of the power line cross-section. These distances of each wire can be calculated as follows:

$$y_k = R_k \sin\varphi_k \tag{10}$$

$$z_k = R_k \cos\varphi_k \tag{11}$$

Then the quadratic moment of the k^{th} wire cross-sectional area $A_k = \pi d_k^2/4$ according the axis y can be calculated by equation (10) and according the axis z by equation (11).

$$I_{yk} = \frac{\pi d_k^{\ 4}}{64} + z_k^{\ 2}\frac{\pi d_k^{\ 2}}{4} \tag{12}$$

$$I_{zk} = \frac{\pi d_k^{\ 4}}{64} + y_k^{\ 2}\frac{\pi d_k^{\ 2}}{4} \tag{13}$$

The polar moment of the wire cross-sectional area to origin of the coordinate system x, y is

$$I_{pk} = I_{yk} + I_{zk} \tag{14}$$

The effective cross-sections of the power line 450AlFe6 parts are: A_{Fe} = 73,89 mm^2, A_{Al} = 445,32 mm^2 and the effective cross-sectional area of the power line is A = 519,21 mm^2. The effective quadratic moments of inertia of the power line cross-sectional area are: $I_z = I_y$ = 28528,3 mm^4.

The effective material properties of the selected single power lines calculated by expressions (1)-(9) are the following:

- the effective electric conductance - $\gamma_L^H = 3.1871\times10^{7}$ Sm^{-1}
- the effective thermal conductance - $\lambda_L^H = 214{,}71\,\mathrm{Wm^{-1}K^{-1}}$
- the effectivethermal expansion coefficient - $\alpha_{TL}^H = 1{,}9743\times10^{-5}\ \mathrm{K^{-1}}$
- the effective elasticity modulus for axial loading - $E_L^{NH} = 89923{,}76\,\mathrm{MPa}$

- the flexural stiffness - $E_L^{M_yH} = E_L^{M_zH} = 97146{,}55\,\mathrm{MPa}$
- the effective shear modulus - $G_{Ly}^H = G_{Lz}^H = 34586{,}06\,\mathrm{MPa}$
- the torsional elasticity modulus - $G_L^{M_xH}(x) = 28090{,}28\,\mathrm{MPa}$
- effective mass density for axial beam vibration- $\rho_L^{NH} = 3432{,}91\,\mathrm{kgm^{-3}}$
- effective mass density for torsional vibration- $\rho_L^{NH} = 2811{,}94\,\mathrm{kgm^{-3}}$

4. Conclusion

In this paper the homogenization of material properties of the AlFe power line has been presented. The advantages of this homogenization approach are that not only effective elasticity modulus for tension and for the bending has been evaluated but also the elasticity modulus for shear and torsion can be calculated. These homogenized material properties than can be used in elastostatic and modal analysis of the power lines by our new beam finite element [9, 10].

Acknowledgement

This paper has been supported by the Slovak Grant Agencies VEGA No. 1/0453/15 and APVV-0246-12 and VEGA No. 1/0228/14.

This contribution is the result of the project implementation: Industrial Research Centre for operating lifetime of selected components of power plants (ITMS: 26240220081), supported by the Research & Development Operational Programme funded by the ERDF.

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