LONG BRAGG GRATING FIBRE OPTIC SENSORS BASED ON OFDR

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1. Introduction

Traditional use of optical fibers for information signal transmission is now broadly widened by their applications in the space distributed sensing of many physical parameters [1], [2]. Due to the rather high sensitivity and space resolution of these sensors their importance is significant and the amount of investment into this field is growing rapidly worldwide [3]. At present many approaches to fiber optic (FO) sensors do exist. Special group of these sensors is based on optical fibres with the inscribed fiber Bragg gratings (FBG) especially with so called long-length FBG (L-FBG) (L≥100 mm) [4]. Usage of these sensors makes possible to get distribution of the measured quantity along the L-FBG with relatively high space resolution. One of the most significant applications of these sensors is e.g. in structural health monitoring (SHM) where local mechanical stress and tension are necessary to measure on-line. Reported spatial resolutions are in mm or sub mm range [5, 6].
In this contribution we focus on the use of L-FBG sensors for the measurement of space distribution of physical quantities along the sensing optical fiber which is based on Optical Frequency Domain Reflectometry (OFDR). The main result is the description how to measure the space distribution of the reflection spectra along the sensing OF and their utilization for the measurement of an external physical quantity. Finally a special case of the measurement of low induced birefringence in L-FBG optical fiber is described.

2. Transmission matrix of the homogeneous Fiber Bragg Grating (FBG) section

The most frequently used FBG are produced by exposing an optical fiber to ultraviolet light beam of sufficient intensity. From the point of view of mode propagation in the fiber core one can consider the FBG as the perturbation of the effective refractive index $\delta n_{eff}(z)$ characterised by the relation

$$\delta n_{eff}(z) = [\delta n_{eff}(z)]_{av}.\{1 + \nu.\cos[(2\pi/\Lambda).z]\} \tag{1}$$

where $[\delta n_{eff}(z)]_{av}$ is the constant averaged effective index value of one FBG period (DC index change), Λ is the designed FBG period and ν is depth of the index modulation (visibility). This perturbation results in the mode coupling between all allowed modes in the fiber and principally influences the transmission and reflection spectra of the BG. If one considers an elementary single mode homogeneous FBG section of length Δz with the forward and backward propagating modes only the relation between the complex amplitudes of forward mode $R(z)$ and backward one $S(z)$ can be described by the matrix equation as follows:

$$\begin{bmatrix} R(0) \\ S(0) \end{bmatrix} = T^{BG} \begin{bmatrix} R(\Delta z) \\ S(\Delta z) \end{bmatrix} \tag{2}$$

where

$$T^G = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} = \begin{bmatrix} \cosh(\gamma_B \Delta z) - j\frac{\sigma}{\gamma_B}\sinh(\gamma_B \Delta z) & -j\frac{\kappa}{\gamma_B}\sinh(\gamma_B \Delta z) \\ j\frac{\kappa}{\gamma_B}\sinh(\gamma_B \Delta z) & \cosh(\gamma_B \Delta z) + j\frac{\sigma}{\gamma_B}\sinh(\gamma_B \Delta z) \end{bmatrix} \quad (3)$$

and $$\gamma_B = \sqrt{(\kappa^2 - \sigma^2)}. \quad (4)$$

σ_g is the „general self-coupling coefficient (DC- coupling coefficient)“ and κ is „alternating coupling (AC) coefficient“. They are given by relations

$$\sigma_g = (2\pi/\lambda).[n_{eff}\,(\delta n_{eff})_{av} - (\pi/\Lambda)] \quad \text{and} \quad \kappa = \kappa^* = (\pi/\lambda).\nu.[\delta n_{eff}]_{av}\ . \quad (5)$$

λ is the optical radiation wavelength in free space and n_{eff} is the effective refractive index of the mode in the fiber core.

The amplitude reflection coefficient of the FBG is defined as ρ= [S(0)/ R(0)]. Using (2), (3) and putting R(0)=1 and S(Δz)=0 the reflection coefficient can be expressed according to [7] as

$$\rho = \frac{S(0)}{R(0)} = S(0) = \frac{T_{21}}{T_{11}} = \frac{-\kappa.\sinh\{[\sqrt{(\kappa^2 - \sigma_g^2)}].\Delta z\}}{\sigma_g.\sinh\{[\sqrt{(\kappa^2 - \sigma_g^2)}].\Delta z\} + i.[\sqrt{(\kappa^2 - \sigma_g^2)}].\cosh\{[\sqrt{(\kappa^2 - \sigma_g^2)}].\Delta z\}} \quad (6)$$

As a result for the evaluation of reflection spectra of the homogeneous elementary FBG section $\boldsymbol{T^G}$ matrix elements can be used.

3. Long length fiber Bragg grating sensor system

The basic idea of the FBG sensor consists in the interaction of FBG with the external physical quantity like strain, temperature, electric field and many others what results in the measurable change of the reflected spectrum from the FBG. To be able to evaluate quantitatively the measured quantity an appropriate model of that interaction is needed.

One of the most important types for the use of the FBG for sensing purposes is the ***"L-FBG"*** in combination with the ***"Optical Frequency-Domain Reflectometry"*** (OFDR). The basic principle of such sensing system can be explained using Fig. 1. The optical signal from the broad-band tuneable laser source TS is launched into 3dB optical fiber coupler C1 which equally splits the optical power into couplers C2 and C3. These are the inputs of two fiber interferometers – first consisting from the C2, mirrors M1, M2 and photodiode PD1 and the second one from C2, M3, L-FBG and photodiode PD2. Signals from the two optical fiber arms in the first interferometer terminated by the mirrors M1 and M2 interfere and the interference signal is detected by the photodiode PD1. The detected power is principally described by the relation

$$\boldsymbol{I_{PD1}} \sim \cos(2n_{eff} L_{REF}\, k) \quad (7)$$

where L_{REF} represents the relative position of M1 to M2. This signal is changing periodically with the changing wave number k and its periodicity can be adjusted mainly by the choice of L_{REF}. The period of the signal is given by

$$\Delta k = \pi/(n_{eff} L_{REF}) \quad (8)$$

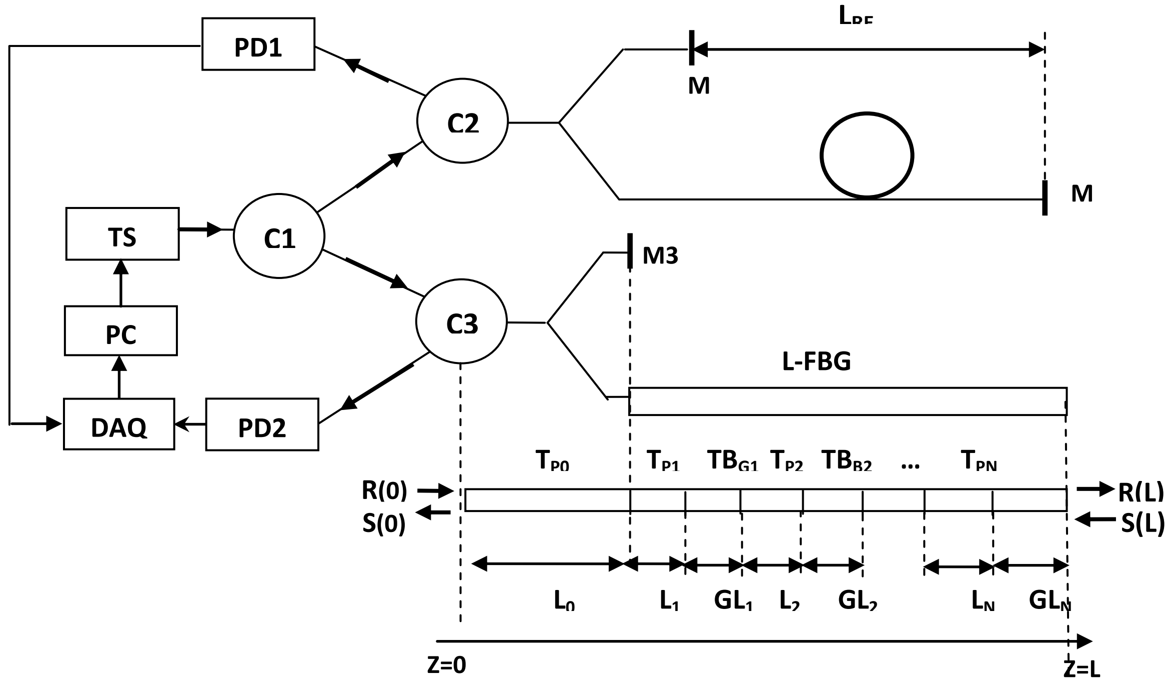

Fig. 1 *The simplified scheme of the L-FBG sensing system based on OFDR*

This signal is used for the equidistant sampling of the measured signal from the second FBG interferometer. It can be done by triggering the analog-to-digital convertor (ADC) in data acquisition device (DAQ).

The second interferometer (the measuring one) is created by C3, M3, L-FBG and PD2. The whole sensing L- FBG is considered to consist from N partial L-FBG sections indexed by "i" = 1 to N. Each partial L-FBG section of the length L_i is created by M_{ij} series of further homogeneous elementary FBG-s with the length „ Δz" and the individual local grating parameters γ_B, σ_g, κ as defined by (4-5). So the length of the particular L-FBG section is $GL_i = (M_{ij}.\Delta z)$. Between particular L-FBG sections the pieces of "connecting single-mode fibers" of the length L_i are placed. The transmission matrices of the elementary homogeneous FBG sections $\boldsymbol{T^G}$ are defined by (3) and the transmission matrix $\boldsymbol{T^P}$ of the inserted connecting piece that really represents the phase shift of the transmitted optical signal only is defined as follows.

$$T_i^P = \begin{bmatrix} e^{-jn_{eff}L_ik} & 0 \\ 0 & e^{jn_{eff}L_ik} \end{bmatrix} \qquad (9)$$

The transmission matrix of the whole L-FBG can now be represented by a proper multiplication of all particular local matrices of the elementary homogeneous FBG-s $\boldsymbol{T^G}$ indexed by "ij" and

the matrices of "connecting sections" $\boldsymbol{T^P}$ indexed by "i" and potentially located between the sections "i" as follows (see Fig. 2).

$$T^G = [T_0^P].[T_1^P].[T_{11}^G].[T_{12}^G] .... [T_{1M1}^G].[T_2^P].[T_{21}^G][T_{22}^G]. ... [T_{2M2}^G].[T_3^P]$$

$$.... [T_N^P].[T_{N1}^G][T_{N2}^G]. ... \left[T_{N(MN)}^G\right]. \qquad (10)$$

As it was stated in (6) having the coefficients T_{11} and T_{21} of the total reflection matrix (10) its total reflection spectrum can be calculated.

The resultant interference signal I_{D2} seen by the diode D2 is given by the sum of the signal $S_{M3}(0)$ reflected from the reference mirror M3 and the $S_{BG}(0)$ reflected from the whole L-FBG. It holds

$$I_{D2} = |S_{M3}(0) + S_{BG}(0)|^2 \quad (11)$$

where $S_{M3}(0) = \exp[i(2n_{eff}.L_0.k+\pi)]$ and n_{eff} is the effective refractive index of the fiber core, L_0 is the distance given in the Fig. 2, k is the wave number in vacuum. Equation (11) indicates that total interference signal seen by D2 represents the sum of the reflecting spectra from the local homogeneous elementary FBG modulated by the corresponding beating frequencies defined by positions of the elementary sections with respect to the M3 position and the fiber effective index of refraction. It is necessary to remind that it is true if the reflectivity of each elementary FBG is less than approx. 10 % and the depth of the refractive index modulation in each elementary FBG is not greater than approx. 0,4 only [7,8]. Under these conditions the total signal at PD2 can be expressed as the sum of the particular modulated local reflection spectra Rij(k)

$$R_{D2} = \sum_{i,j} R_{ij}(k).\cos(2n_{eff}.z_{ij}.k) = \sum_{i,j} R_{ij}(k).\cos(\omega_{ij}.k) \quad (12)$$

Each component $R_{ij}(k)$ is modulated by harmonic beating signal of the frequency ω_z that is unambiguously defined by the position of the particular elementary FBG as follows

$$\omega_z = 2n_{eff}.z \quad (13)$$

Equation (12) is the crucial for the processing of the signal measured by the photodiode PD2. If one now applies the „Short Time Fourier Transform“ (STFT) procedure to the measured total spectrum R_{D2} and $\omega_z=\omega_{z1}=2n_{eff}.z_1$ is constant he can obtain the reflection spectra corresponding to the location $z=z_1$. In this way by repeating this procedure for all positions corresponding with all elementary FBGs one can get the complete information about all reflection spectra distributed along the L-FBG.

4. L-FBG sensor for the measurement of low birefringence in optical fibers

In the following let us consider a special case when we have the whole totally homogeneous L-FBG (e.g. we have only one section from the above L-FBG) and all elementary FBG sections with the length Δz are the same and some physical quantity like lateral stress is applied along the grating what results in the appearance of the homogeneous birefringence along the FBG. Let us consider two principal polarization modes – slow and fast with the corresponding effective refractive indices n_{effs}, n_{efff} - propagating in forward and backward direction in the L-FBG. As a result two reflection spectra appear. When the birefringence is significant then two different shifted reflection spectra can be easily observed.

The difference between the reflection spectrum maxima wavelengths is given as $\Delta\lambda_{fs} = \lambda_f - \lambda_s = 2\Lambda(n_{effs}-n_{efff})$. ***Consequently by measurement of $\Delta\lambda_{fs}$ one can measure the local birefringence of optical fibre.*** However when the birefringence is rather small the both maximum wavelengths are very near each other and cannot be clearly distinguished. In this case the reliable measurement is not possible. Here another approach can be applied.

As it follows from the equation for the resulting L-FBG spectrum (12), in a special case when the

L-FBG is totally homogeneous and the low value birefringence is constant along the whole L-FBG the resulting spectrum can be described more simply as follows

$$R_{D2} = R(k).\sum_j \cos(2\,n_{eff}.z_j.k) = R(k)\int_{z_0}^{z_1} \cos(2\,n_{eff}.z.k)dz = R(k).C(k) \quad (14)$$

The integral in the sum is denoted as *C(k)*. The simplification consists in taking into account that reflection spectra of all elementary FBG are equal and therefore in the sum only the beating signals remained. Finally the summation can be replaced by the integration between the z_0, z_1 that is between the beginning and the end of the FBG.

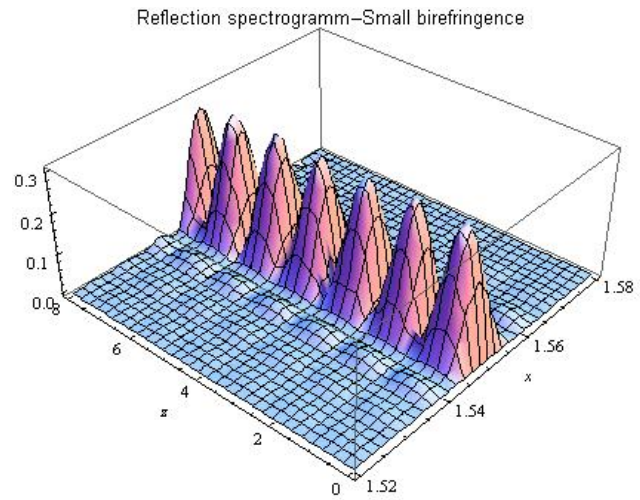

Fig. 2 *Example of L-FBG spectrogram for small birefringence, $\Delta n_{sf} = n_s - n_f = 0.00028$*

Now the power profile of the signal on the PD2 can be expressed as the Fourier transform of the (14) as follows:

$$I_{D2} = |F\{R(k).C(k)\}(\omega_z)| = |F\{\omega_z\}| \quad (15)$$

As it is generally known the Fourier transform of the product of two functions can be equally replaced by the convolution of the corresponding Fourier transforms of these functions. So it holds

$$I_{D2} = |F\{R(k)\}(\omega_z) * F\{C(k)\}(\omega_z)| \quad (16)$$

When the birefringence is taken into account then Bragg spectrum can be expressed as the sum of the spectra of the fast and slow mode respectively. So we can write

$$R_b(k) = R_s(k) + R_f(k) = R_s(k-k_0) + R_f(k) \quad (17)$$

where the k_0 is the maxima separation wavenumber between the fast and slow mode reflection spectra. It can be expressed as

$$k_0 = \frac{2\pi}{\lambda_{Bf}} - \frac{2\pi}{\lambda_{Bs}}. \quad (18)$$

λ_{Bf}, λ_{Bs} are the Bragg wavelengths of the fast and slow modes respectively. Implicitly we have considered that the amplitudes in the spectral maximum points of both modes are the same and also that no mode coupling between modes along the fiber does exist. The Fourier transform of the birefringent spectrum (17) can be written in the form

$$F\{R_b(k)\}(\omega_z) = F\{R_s(k) + R_s(k-k_0)\}(\omega_z) = \left(1 - e^{-j\omega_z k_0}\right)F\{R_s(k)\}(\omega_z) \quad (19)$$

Now the power profile I_{D2} can be expressed as follows

$$\left|\left(1 - e^{-j\omega_z k_0}\right)F\{R_s(k)\}(\omega_z) * F\{C(k)\}(\omega_z)\right|. \quad (20)$$

Except others this signal contains also the term given by absolute value of the convolution $|\exp(-j\omega_z k_0) * F\{C(k)\}(\omega_z)|$. It results in the beating of the power profile which can be expressed as a function of "z" coordinate as follows

$$I_{D2-beating} = \left(\frac{1}{k_0}\right)[1 - \cos\{2n_{eff}(z - z_0)k_0\}]^{0.5}. \quad (21)$$

Using (21) we can conclude - the beating component of the signal oscillates linearly with the position z and the oscillation period Π_z is inversely proportional to k_0. It holds

$$\Pi_z = \frac{\pi}{n_{eff}\,k_0} \quad (27)$$

So in such a way by measurement of oscillation period Π_z one can evaluate the separation of the Bragg gratings maxima k_0 of the fast and slow modes respectively and consequently also the low birefringence of the fiber. An example of the simulation of low birefringence spectrogram of L-FBG using the model described above is given in the Fig. 2 . The beating period is approx. 1.8 mm and the corresponding birefringence is $\Delta n_{eff-sf} = n_{eff-s} - n_{eff-f} =$ 0.000277.

5. Conclusion

The L-FBG based optical fiber sensor is an efficient tool for the measurement of the space distribution of several physical quantities. Some of them can be transformed into the induced birefringence in the optical fiber with the inscribed L-FBG. This can be identified by the measurement of the corresponding changes in the L-FBG reflection spectra. There exist direct relation between the maximum wavelength shift in the reflection spectra of the L-FBG and the induced birefringence. The reflection spectra changes can be extracted by the OTDR signal processing using STFT approach. However for rather small birefringence values there is a problem with the identification of the maximum shift in the reflection spectra which is very small and difficult to evaluate. The solution in this situation is the measurement of the power reflection spectra oscillation period along the grating. It was shown that the cycling period is unambiguously coupled with the induced birefringence in the fiber.

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