

# MODELLING THE ANISOTROPY OF MAGNETIC FIELD ANNEALED Fe<sub>61</sub>Co<sub>19</sub>Si<sub>5</sub>B<sub>15</sub> AMORPHOUS ALLOY

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## 1. Introduction

In spite of the fact, that main ideas of Jiles-Atherton model was first presented for isotropic materials in 1983 [1], this model is still intensively developed. Two most important steps forward were done in 1996, when Sablik and Jiles presented possibility of considering the stresses and magnetostriction in this model [2], as well as when Ramesh extension of the model enabled anisotropic materials modelling [3, 4].

However, practical implementation of Jiles-Atherton model with both of these extensions is sophisticated. First of all, identification of extended model's parameters on the base of experimental data is not obvious and require application of cognitive algorithms, such as evolutionary strategies [5]. This is time consuming and results exhibit uncertainty, due to the stochastic character of evolutionary strategies.

Paper presents the proposition of step towards overcoming these problems. Due to the fact, that stress dependence of magnetic hysteresis loop in extended Jiles-Atherton model may be reduced to influence of stresses on magnetic anisotropy, modelling may be simplified to the model of anhysteretic magnetization. This significantly reduces calculation time, decreases uncertainty of calculation as well as creates novel possibility of analyses of changes of average anisotropy energy density under the stresses.

## 2. Magnetic uniaxial anisotropy energy in Jiles-Atherton model with Ramesh extension

Conception of Jiles-Atherton model is based on idea of anhysteretic magnetization  $M_{ah}$ . Such magnetization would be observed, if material would be successfully demagnetized by alternating magnetic field, under constant value of magnetization field (polarized demagnetization). However, accurate measurement of flux density  $B$  during such demagnetization is very sophisticated. As a result the anhysteretic magnetization characteristics are rather not presented in the literature.

In Ramesh extension of Jiles-Atherton model, this magnetization is given as a weighted sum of isotropic anhysteretic magnetization  $M_{ah\_iso}$  and anisotropic anhysteretic magnetisation  $M_{ah\_aniso}$ [3, 4]:

$$M_{ah} = (1-t) \cdot M_{ah\_iso} + t \cdot M_{ah\_aniso} \quad (1)$$

where  $t \in \langle 0, 1 \rangle$  describes the participation of anisotropic phase in the material. In Jiles-Atherton model isotropic anhysteretic magnetization  $M_{ah\_iso}$  is given by the Langevin equation [1]:

$$M_{ah\_iso} = M_s \left[ \coth \left( \frac{H_e}{a} \right) - \left( \frac{a}{H_e} \right) \right] \quad (2)$$

where parameter  $a$  (given in A/m) describes the domain walls density,  $M_s$  is saturation magnetization and  $H_e$  is effective magnetic field considering interdomain coupling  $\alpha$ .

In Ramesh extension of Jiles-Atherton model, the anisotropic anhysteretic magnetisation  $M_{ah\_iso}$  is given by the following equations [3, 4]:

$$M_{ah\_aniso} = M_s \left[ \frac{\int_0^\pi e^{E(1)+E(2)} \sin\theta \cdot \cos\theta \cdot d\theta}{\int_0^\pi e^{E(1)+E(2)} \sin\theta \cdot d\theta} \right] \quad (3)$$

$$E(1) = \frac{H_e}{a} \cos\theta - \frac{K_{an}}{M_s \cdot \mu_0 \cdot a} \sin^2(\psi - \theta) \quad (4)$$

$$E(2) = \frac{H_e}{a} \cos\theta - \frac{K_{an}}{M_s \cdot \mu_0 \cdot a} \sin^2(\psi + \theta) \quad (5)$$

where  $K_{an}$  is the average energy density connected with uniaxial anisotropy in magnetic material, and  $\psi$  is the angle between direction of magnetizing field and the easy axis of magnetization due to the anisotropy.

However, typing mistake occurs in equation (3) in the original paper [4]. Analysis of physical principles presented in the papers [3, 4], as well as observation of coherence with Langevin equation for  $K_{an} = 0$  (which is obvious for isotropic material) lead to the conclusion, that correct form of anisotropic anhysteretic magnetisation  $M_{ah\_iso}$  should be given by the following equation:

$$M_{ah\_aniso} = M_s \left[ \frac{\int_0^\pi e^{\frac{E(1)+E(2)}{2}} \sin\theta \cdot \cos\theta \cdot d\theta}{\int_0^\pi e^{\frac{E(1)+E(2)}{2}} \sin\theta \cdot d\theta} \right] \quad (6)$$

In original Jiles-Atherton model the hysteresis is modelled by considering of the sign of changes of magnetizing field in differential equation describing non-reversible magnetisation  $M_{non\_r}$ , as a part of total magnetization  $M$  of the material. However, non-reversible part of magnetization is not connected with magnetic anisotropy [1, 2, 4]. As a result non-reversible magnetisation  $M_{non\_r}$  don't have to be considered for modelling of stress-induced anisotropy.

Uniaxial, stress induced anisotropy  $K_{an}$  is given by the following equation [2]:

$$K_{an} = \frac{3}{2} \lambda_s \sigma \quad (7)$$

where  $\lambda_s$  is saturation magnetostriction and  $\sigma$  are stresses applied to the core. In such a case, the tensile stresses are marked as positive, and compressive stresses are marked as negative. Moreover, during the analysis, it should be considered, that value of saturation

magnetostriction  $\lambda_s$  changes under stresses [6]. If magnetizing field  $H_e$  in the core is perpendicular to the stresses  $\sigma$ , effective stresses  $\sigma_e$  may be calculated as follows [7]:

$$\sigma_e = -\nu \cdot \sigma \quad (8)$$

where  $\nu$  is the Poisson ratio.

### 3. Experimental data

Measurements of the influence of compressive stresses on magnetic hysteresis loop was carried out using digitally controlled hysteresisgraph and special nonmagnetic backings. Uniform compressive stresses were applied by special mechanical device described elsewhere [7].

The investigation of the stress dependence of magnetic hysteresis loop was performed on the ring-shaped cores made of  $\text{Fe}_{61}\text{Co}_{19}\text{Si}_5\text{B}_{15}$  amorphous alloy. External diameter of cores was 32 mm, internal diameter was 25 mm, while their height was equal 8 mm. Core was annealed in  $380^\circ\text{C}$  for 1 h. To induce perpendicular anisotropy in the ring-shaped core, cores were subjected to magnetizing field equal 260 kA/m during the annealing. This magnetizing field was generated perpendicularly to the base of the ring-shaped core.

### 4. Implementation of the model and method of determination of model's parameters

Jiles-Atherton model with Ramesh extension was implemented in MATLAB. For integration necessary to solve the equation (6), the Gauss-Kronrod approximation was used [8]. Parameters of Jiles-Atherton models were identified during the optimisation process, where target function was given as a sum of squares of differences between experimental results and the results of modelling. For minimisation of the target function, simplex search method of Lagarias et al. was applied [9].

It should be indicated, that unphysical values of average anisotropy energy density  $K_{an}$  lower than zero may be achieved during the magnetization process. In such a case, the direction of anisotropy easy axis have to be verified considering equations (4) and (5).

Quantitative analysis indicated, that in presented results isotropic phase may be modelled as about 20% of semi-hard phase with constant relative permeability  $\mu_{iso}$  equal 4928. As a result  $M_{ah\_iso}$  was described as  $\mu_0 \cdot \mu \cdot H_e$ .

### 5. Results

Results of determination of parameters of Jiles-Atherton model with Ramesh extension for magnetic are presented in the table 1. The results of measurements of the influence of compressive stresses  $\sigma$  on the shape of hysteresis loop of  $\text{Fe}_{61}\text{Co}_{19}\text{Si}_5\text{B}_{15}$  amorphous alloy annealed in the perpendicular magnetizing field equal 260 kA/m together with the results of modelling of anhysteretic magnetization are presented in figure 2a. Figure 2b presents changes of average anisotropy density  $K_{an}$  under the compressive stresses.

Tab. 1. *Jiles-Atherton model parameters identified for hysteresis loop of  $\text{Fe}_{61}\text{Co}_{19}\text{Si}_5\text{B}_{15}$  amorphous alloy subjected to compressive stresses  $\sigma$  equal -7 MPa.*

$M_s$	$a$	$\alpha$	$K_{an}$	$t$	$\mu_{iso}$
A/m	A/m	-	J/m <sup>3</sup>	-	-
$6.44 \cdot 10^5$	19.9	$7.07 \cdot 10^{-6}$	1008	0.796	4928

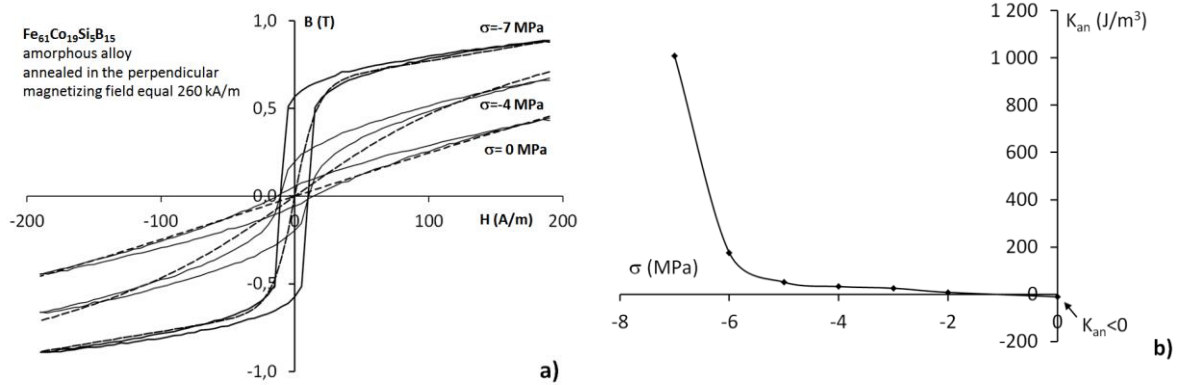


Fig.1: *Compressive stress  $s$  dependence of: a) shape of measured  $B(H)$  hysteresis loop (solid line) together with simulated anhysteretic magnetization (dotted line), b) compressive stress  $\sigma$  dependence of average anisotropy energy density  $K_{an}$  (for easy magnetization axis considered in direction of magnetization).*

## 6. Conclusions

Results presented in figure 1a indicate, that anhysteretic magnetization curve given by equation (6) is in good agreement with experimental results for significant uniaxial anisotropy in magnetic material. For this reason, results of modelling are in good agreement with experiments for compressive stresses equal 7 MPa (strong stress induced anisotropy) or for 0 MPa (unambiguous magnetic field annealing anisotropy). For the mixed anisotropy (medium value of stresses) agreement between modelling and experiment is much worse. Moreover, when easy axis of magnetization is perpendicular to assumed easy axis, negative values of anisotropy can be achieved. This indicate that change of assumed direction of easy axis of magnetization is required.

Presented results indicate, that corrected Jiles-Atherton model extended by Ramesh may be successfully used for determination of average density of uniaxial anisotropy energy on the base of  $B(H)\sigma$  magnetoelastic characteristics.

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