TWO-GROUP NUMERICAL ANALYSIS OF ONE DIMMENSIONAL TABLE NUCLEAR REACTOR

Filip Osuský[1], Branislav Vrban[1], Štefan Čerba[1], Jakub Lüley[1], Ján Haščík[1]

[1] *Slovak University of Technology in Bratislava, Institute of Nuclear and Physical Engineering, Ilkovičova 3, 812 19 Bratislava, Slovakia*

E-mail: xosuskyf@stuba.sk, branislav.vrban@stuba.sk, stefan.cerba@stuba.sk, jakub.luley@stuba.sk, jan.hascik@stuba.sk

Received 08 May 2014; accepted 20 May 2014

1. Introduction

Due to the complexity of Boltzmann transport equation, for the computational modelling there is a need to simplify operators in the equation. One well known approximation is diffusion theory, which is often used for numerical calculations. This approximation describes the migration of neutrons in space. It also eliminates few challenging variables, such as neutrons angular distribution and in our case energy variability (only two energy groups of neutrons are considered). This simplification has also its limitations and the diffusion theory is applicable in the environment without large local absorbers [1]. The limitations are: environment is homogeneous, steady state type of analysis ($\partial\varphi/\partial t = 0$), neutron scattering is isotropic. The influence of heterogeneousness is decreasing with larger migration area of neutrons and therefore the diffusion theory can be used instead of transport theory specially in fast reactor systems.

2. Numerical approximation

For the purpose of calculation we have developed whole code in C++. This code solves the neutron diffusion equation Eq.(1) [1].

$$D_g \Delta\phi_g - \Sigma_{a,g}\phi_g - \sum_h \Sigma_{g\to h}\phi_g + \sum_h \Sigma_{h\to g}\phi_h + \chi_g \sum_h \nu\Sigma_{f,h}\phi_h = 0 \tag{1}$$

where g represents the group. The first term represents the leakage, the second term represents absorptions, third represents disappearances from the group by transfer to another group (h), fourth represents contributions by transfer from another group and last term represents contribution of fission reaction to specified energy group (χ_g is the proportion of neutrons emitted by fission in group g, ν is average number of neutrons produced after fission). The program solves Eigenvalue problem Eq.(2) by the use of iterative Power method described in Eq.(3) [2].

$$M\phi = \lambda F\phi = \frac{1}{k}F\phi \Rightarrow M^{-1}F\phi = A\phi = k\phi \tag{2}$$

where k is the largest Eigenvalue corresponding to ϕ - fundamental eigen vector.

$$\phi^{(l)} = A^l\phi^{(0)} \tag{3}$$

Scaling is needed when $k \neq 1$. Infinity norm scaling is used in this program Eq.(4).

$$\phi^{(l)} = \delta_l A^l\phi^{(0)} \tag{4}$$

where δ_l is scaling factor and it is inverted maximum value of fundamental eigen vector. Convergence can be shown for random vector, that is not equal to zero Eq.(5).

$$\phi^{(0)} = c_1 u_1 + c_2 u_2 + \cdots\cdots + c_N u_N \tag{5}$$

where u_i is the i[th] normalized eigen vector of matrix A and is corresponding to k_i (harmonics of eigenvalue). Then Eq.(3) can be written as Eq.(6) [2].

$$\phi^{(l)} = A^l\left(c_1 u_1 + c_2 u_2 + \cdots\cdots + c_N u_N\right) = c_1 \cdot k_1^l u_1 + c_2 \cdot k_2^l u_2 + \cdots\cdots + c_N \cdot k_N^l u_N \tag{6}$$

The terms k_i/k_l in in Eq.(7) are negligible (for $i > 1$) after enough multiplications steps (l) and the obtained result consists of fundamental eigenvalue with corresponding eigen vector.

$$\phi^{(l)} = k_1^l \left(c_1 u_1 + \sum_{i=2}^{N} \left[\left(\frac{k_i}{k_l} \right)^l c_i u_i \right] \right) \approx k_1^l c_1 u_1 \tag{7}$$

The Box Scheme was used for discretization of examined area (Fig. 1).

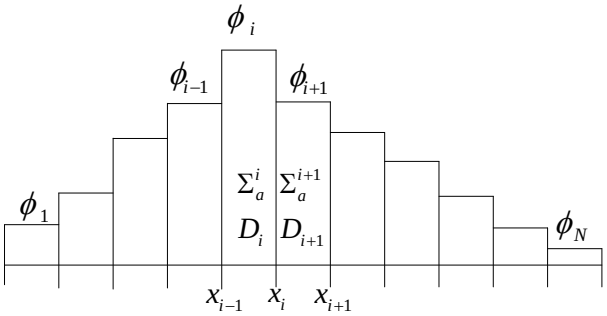


Fig.1: *Description of Box Scheme variables*

Therefore the equation Eq.(1) can be written as Eq.(8) for two groups of neutrons.

$$-\tilde{D}_g^{k-1}\phi_g^{k-1} + \left(\tilde{D}_g^{k-1} + \tilde{D}_g^k + \Sigma_{rg}^k h_k\right)\phi_g^k - \tilde{D}_g^k \phi_g^{k+1} = \lambda \chi_g^k h_k \left(\sum_{g'=1}^{2} \nu\Sigma_{fg'}^k \phi_{g'}^k \right) + h_k \left(\sum_{\substack{g'=1 \\ g' \neq g}}^{2} \Sigma_{s,g'g}^k \phi_{g'}^k \right) \tag{8}$$

where Σ_r represents the removal macroscopic cross section, h_k the length of interval, λ fundamental eigenvalue, χ_g the proportion of neutrons emitted by fission in group g, Σ_f the fission macroscopic cross section, Σ_s the scattering macroscopic cross section between energy groups of neutrons and $\tilde{D}$ coupling coefficient. Removal macroscopic cross section consist of absorption macroscopic cross section and scattering macroscopic cross section into another group Eq.(9). And coupling coefficient is shown in Eq.(10) [2].

$$\Sigma_{rg}^k = \Sigma_{ag}^k + \sum_{\substack{g'=1 \\ g' \neq g}}^{2} \Sigma_{s,gg'}^k \tag{9}$$

where Σ_a represents the absorption macroscopic cross section.

$$\tilde{D}_g^k = 2\frac{\dfrac{D_g^k}{h_k}\dfrac{D_g^{k+1}}{h_{k+1}}}{\dfrac{D_g^k}{h_k} + \dfrac{D_g^{k+1}}{h_{k+1}}} \tag{10}$$

where D represents diffusion coefficient.

3. Material analysis

It is necessary to note that this is only theoretical approximation and that is mainly because the values of macroscopic scattering cross section were simplified with certain degree of uncertainty. Geometry is also simplified into one dimension and some dimensions are changed for faster convergence of numerical calculation (Fig. 2.).

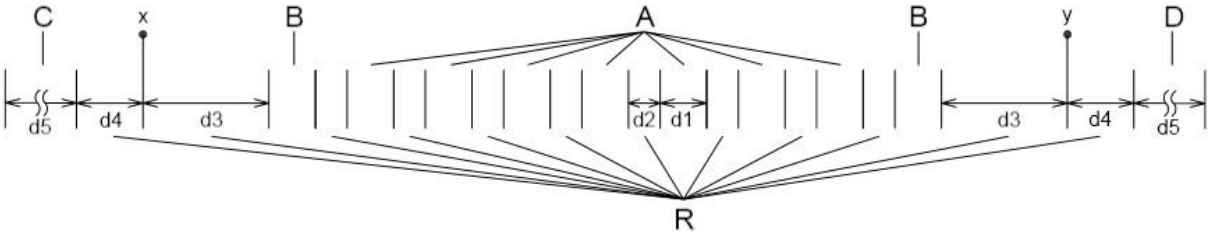


Fig.2: *Geometry*

where material A represents UO_2 with enrichment 5% (dimension *d1* is 7 mm and is same for every material A and B), B is UO_2 with enrichment 4.4% and with 3.35% of ^{153}Gd, C is B_4C (dimension *d5* is 142 mm), R stands for H_2O (dimension *d2* is 5 mm, *d3* 19 mm, *d4* 10 mm). D is mixture of whole area between x – y points (*d5* is 142 mm).

All cross sections are obtained from ENDF/B-VII.0 library for temperature 300 K and for the energies 0.0253 eV and 2 MeV [3] (see Tab. 1. and Tab. 2.).

Tab. 1. *Material properties for neutron energy 0.0253 eV.*

	UO_2(5%)	UO_2Gd(4.4%)	H_2O	B_4C	Mixture
Σ_a [cm^{-1}]	0.00236	0.258839024	0.02198	0.04003	0.03868
$\nu\Sigma_f$ [cm^{-1}]	0.00927	0.008448208	0	0	0.00406
D [cm]	19.46342	1.221024339	0.15510	0.00109	0.27279

Tab. 2. *Material properties for neutron energy 2 MeV.*

	UO_2(5%)	UO_2Gd(4.4%)	H_2O	B_4C	Mixture
Σ_a [cm^{-1}]	1.52E-05	1.69699E-05	2.4E-06	1.3E-07	8.3E-06
$\nu\Sigma_f$ [cm^{-1}]	0.000179	0.000178005	0	0	8E-05
D [cm]	101.9692	99.67455265	1.37155	1.53263	2.45256

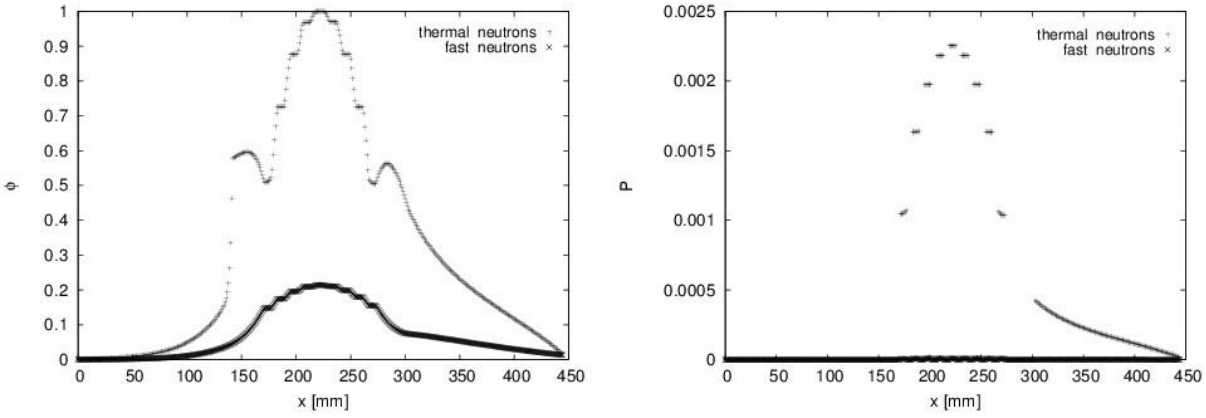
One possible way for the calculation of diffusion coefficient is the undermentioned equation Eq.(11) [4]. This is applicable if isotropic scattering is used.

$$D \cong \lambda_{tr}\frac{1}{3} = \frac{1}{3\Sigma_{tr}} = \frac{1}{3\left(\Sigma_t - \bar{\mu}_0\Sigma_s\right)} \approx \frac{1}{3\Sigma_t} \qquad (11)$$

where Σ_t stands for the total macroscopic cross section and $\bar{\mu}_0$ unsymmetrical parameter of neutron scattering (if isotropic scattering is applied $\bar{\mu}_0 = 0$) [4].

In this case the molecular cross section of H_2O is not considered, even if the molecular cross section is higher than calculated cross sections of particular nuclides [4]. The proportional distribution of neutron flux is shown in the Fig. 3. *a)*. B_4C is located on the left part of Fig. 3. *a)* and B_4C is good absorber (area until x = 142 mm). The neutron flux decreases almost to the zero in this area. The H_2O is good reflector and also good moderator, so the main increases or decreases of neutron flux are located in the H_2O part of nuclear system for both neutron energy groups. Because the UO_2 fuel has small macroscopic scattering cross section and big diffusion coefficient in comparison with H_2O for both fuel types, it seems that the neutron flux is constant

within the fuel (there are only small changes in distribution of neutron flux within the fuel). Steady decrease of neutron flux is located within uniform mixture. This is caused by the uniformity of the mixture and by the zero incoming boundary condition which assumes vacuum environment behind the boundary (neutrons are not scattered back to the fuel when they cross boundary with vacuum). The power generation is affected mainly by thermal neutrons (Fig. 3. *b)*). This is caused by macroscopic fission cross section which reaches the biggest value for neutrons with thermal energy in the feasible material. This nuclear system is subcritical with k_{eff} = 0.2863.



a) *b*)

Fig.3: *a) Proportional distribution of neutron flux, b) Proportional distribution of power generation*

4. Conclusion

Diffusion theory is very suitable for fast calculations and it is used during development of theoretical reactor design. The successful application of program code was demonstrated. It has also its limitations but it provides relevant information when these limitations are taken into account by researcher. Development of such program codes helps to increase knowledge level and it directly influences nuclear safety. It also allows solving specific issues which are irresolvable by commercial codes. In the future, we have a plan to enlarge geometry into 2 dimensions.

Acknowledgement

This work was financially supported by grant of Science and Technology Assistance Agency No. APVV-0123-12.

References:

[1] P. Reuss: Neutron Physics, EDP Sciences, Ulis France (2008)
[2] J. Han-gyu: I. Solution of One-Dimensional, One-Group Neutron Diffusion Equation (Lectures note 1), NJE, 2008
[3] Online Plotter for MCNP and ENDF cross section data http://atom.kaeri.re.kr/cgi-bin/endfplot.pl
[4] J. Lamarsh: Introduction to Nuclear Reactor Theory, American Nuclear Society, North Kensington USA (2002).