EFFECTIVE MATERIAL PROPERTIES CALCULATION OF THE FGM BAR STRUCTURES

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1. Introduction

One important goal of mechanics of heterogeneous materials is a derivation their effective properties from the knowledge of the constitutive laws and complex micro-structural behaviour of their components. Microscopic modelling expresses the relation between the characteristics of the components of the composite and the average (effective) properties of the composite. In case of the Functionally Graded Material (FGM) it is the relation between the characteristics of the components and the effective properties of the FGM.

The methods based on the homogenization theory (e.g. the mixture rules [1], [2]; self-consistent methods [3]) have been designed and successfully applied to determine the effective material properties of heterogeneous materials from the corresponding material behaviour of the constituents (and of the interfaces between them) and from the geometrical arrangement of the phases. In this context, the microstructure of the material under consideration is basically taken into account by representative volume element (RVE).

Mixture rules are one of the methods for micromechanical modelling of heterogeneous materials. Extended mixture rules [4] are based on the assumption that the constituents volume fractions (formally denoted as fibres – *f* and matrix – *m*) continuously vary as the polynomial functions: $v_f(x,y,z)$ and $v_m(x.y,z)$. The condition $v_f(x,y,z)+v_m(x,y,z)=1$ must be fulfilled. Appropriated material property distribution in the real FGM is then

$$p(x,y,z)=v_f(x,y,z)p_f(x,y,z)+v_m(x,y,z)p_m(x,y,z) \tag{1}$$

Here, $p_f(x,y,z)$ and $p_m(x,y,z)$ are the spatial variations of material properties of the FGM – constituents. The extended mixture rule (1) can be analogically used also for FGM material made of more than two constituents. The assumption of the polynomial variation of the constituent's volume fractions and material properties enables an easier establishing of the main appropriated field equations and allows the modelling of many common realizable variations.

In the literature and in the practical applications mostly the one directional variation of the FGM properties is presented. An exponential law for variation of the constituents volume fractions is very often presented, e.g. in [5], [6], [7] and in many other references.

By the FGM bars (link, beam or rode) the transversal variation (continuously or discontinuously, symmetrically or asymmetrically) has been mainly considered. The more complicated case is when the material properties vary in tree directions - namely in transversal, lateral and longitudinal direction of the FGM bar.

In the contribution, the homogenization techniques of spatial varying (continuously or discontinuously and symmetrically in transversal and lateral direction, and continuously in longitudinal direction) material properties for the FGM bar with selected doubly-symmetric cross-sections are considered. The techniques (the extended mixture rules (EMR), the multilayer method (MM) and the direct integration method (DIM)) are proposed for derivation of the effective elasticity modules for axial loading, the transversal and lateral bending, the shear modules for transversal and lateral shear and uniform torsion, the effective mass density, the electric and thermal conductivities and the thermal expansion coefficient by Their detailed description is given in [8], [9] and [10]).

2. Effective material properties calculation

In design of the bar structures several types of cross-section are used. Especially, for the Micro-Electro-Thermo-Mechanical Systems (MEMS), which have a form like the link, rode or beam structures, the circular, cylindrical and rectangular or hollow cross-sections are most preferable. If the material is homogeneous, the numerical multiphysical analysis (electro-thermal-structural) can be done with homogeneous finite elements. In the last time the inhomogeneous composite materials, like the FGM, have been used to improve the functionality of the systems. The effective material properties of the beams and links depend of the cross-section area type and of the real variation of material properties trough the bar volume. The homogenization of the varying material properties is made from the assumption, that the electric conductance, the thermal conductance, the thermal expansion, the stiffness (axial, flexural, torsional) of real bar is equal to these properties for homogenized bar. The homogenization methods (EMR and MM)are used in the following numerical experiment, were the modal analysis of an actuator like a clampedbeam with spatially varying material properties isdone.

3. Numerical experiment

The clamped FGM beam with rectangular hollow cross-sectionhas been considered (as shown in Fig.1). Its geometry is given with: h_1 = 0.005 m, h_n = 0.00375 m, b_1 = 0.01 m, b_n = 0.0075mand L = 0.1 m. The cross-sectionalarea is $A = 2.1875\times10^{-5}$ m^2; the area moments of inertia are $I_y = 7.12077\times10^{-11}$ m^{4}and $I_z = 2.84831\times10^{-10}$ m^4, the cross-sectional area polar moment of inertia is $I_p = I_y + I_z = 3.56038\times10^{-10}$ m^{4}and the torsion constant is $I_T = 1.6748\times10^{-10}$m^4.

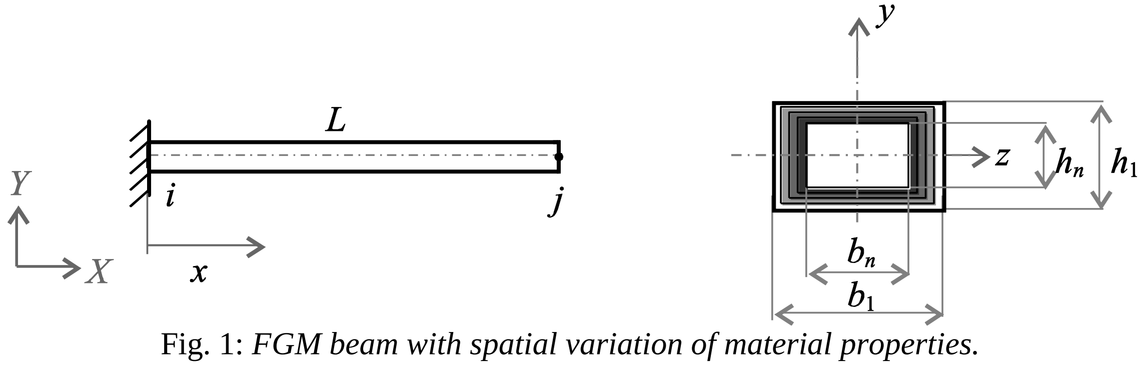

Fig. 1: *FGM beam with spatial variation of material properties.*

The beamis made of a mixture of two components: aluminum Al6061-TO as a matrix (denoted by index *m*) and titanium carbide TiC as a fibre (denoted by index *f*).Material properties of the components are assumed to be constant and their values are: aluminum Al6061-TO – the elasticity modulus $E_m = 69.0$ GPa, the mass density $\rho_m = 2700$ kgm^{-3}, the

Poisson's ratio $\nu_m = 0.33$; titanium carbide TiC – the elasticity modulus $E_f = 480.0$ GPa, the mass density $\rho_f = 4920$ kgm^{-3}, the Poisson's ratio $\nu_f = 0.20$.

The fibre volume fraction varies linearly and symmetrically according to the *x*-*y* and *x*-*z* planes: $v_f \in \langle 0; 1.0 \rangle$ - the inner edges of the beam are made from pure matrix and linearly varies to the outer edges that are made from pure fibre. A constant effective material properties are considered in the *x* – direction.

Using EMR and MM[9], [10]the effective elasticity modulus for axial loading E_L^{NH} in [GPa], for bending about axis *y* - $E_L^{M_y H}$ and about axis *z* - $E_L^{M_z H}$ in [GPa], the shear moduli G_{Ly}^H and G_{Lz}^H in [GPa], torsional shearmodulus $G_L^{M_x H}$ in [GPa] and mass density ρ_L^H in [kgm^{-3}] have been calculated. The influence of the number of divisions *n* to the layers on the homogenized material properties are shown in the Tab. 1.

Tab. 1.*Influence of the number of divisions n to the layers on the homogenized material properties.*

layers *n*	E_L^{NH}	$E_L^{M_y H} = E_L^{M_z H}$	$G_{Ly}^H = G_{Lz}^H$	$G_L^{M_x H}$	ρ_L^H
2	281.839	296.151	112.716	120.614	3849.643
5	283.894	302.229	113.901	124.066	3860.743
10	284.188	303.098	114.071	124.561	3862.328
15	284.242	303.259	114.102	124.653	3862.222
20	284.261	303.315	114.113	124.685	3862.725

The FGM actuator – as a bar clamped at the node *i* has been studied by modal analysis. The first eighteigenfrequencies*f* [Hz] have been found (see Tab.2) using the new FGM beam finite element [10] (calculation has been done with software Mathematica [11] and homogenized material properties for *n*=20 have been applied). Only one our new FGM finite element has been used. The same problem has been solved using a very fine mesh – 8967of SOLID186 elements of the FEM program ANSYS [12]. The average relative difference Δ [%] between eigenfrequencies calculated by our method and the ANSYS solution has been evaluated.

Tab.2.*Eigenfrequencies of the FGM beam.*

Eigenfrequency *f* [Hz]		New Finite Element	Ansys	Δ [%]
1st	flexural about axis *y*	891.6	890.7	0.10
2nd	flexural about axis *z*	1766.6	1759.2	0.42
3rd	flexural about axis *y*	4879.3	5384.1	1.77
4th	torsional	9742.4	9788.3	0.47
5th	flexural about axis *z*	9968.9	9996.0	0.27
6th	flexural about axis *y*	14406.7	14270.0	0.96
7th	axial	21444.6	21473.0	0.13
8th	flexural about axis *z*	25130.9	24420.0	2.91

Very high effectiveness and a good accuracy of our approach can be observed from the results comparison.

4. Conclusions

The homogenization methods (EMR, MM) are used in the numerical experiment, were the modal analysis of clamped beam with spatially varying material properties was done. The homogenization methods can be very effectively used by homogenization of electric, thermal and mechanical material properties of FGM materials which are used in design of the MEMS (which have a form of a simple bar or bar structure). Our solution is very effective and sufficiently accurate comparing to the solution obtained by solid finite elements with layer wise constant material properties [12].

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