INVESTIGATION OF MODULATION INSTABILITY IN THE CASE OF SUPER-GAUSSIAN AND SOLITON PULSES

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Received 30 April 2014; accepted 12 May 2014

1. Introduction

The aim of this article is to investigate the impact of changing its own phase modulation through the chirp parameter on the shape of the input solitary pulse. We want to find the conditions under which the modulation instability may occur in the case of input super-Gaussian (SGP) and solitary pulse. The modulation instability can occur in two cases and it is a possible mechanism for the generation of rogue waves [1]. In the first case it is caused by a combination of dispersion and nonlinear phenomena, in the second case it may be a competition between loss and gain [2]. In the context of optical fibers, modulation instability can be observed in anomalous dispersion regime and can by responsible for a breakup of the CW or quasi- CW radiation into a train of ultrashort pulses [3].

We have simulated the super-Gaussian pulses and then the soliton waves propagating in the optical fiber by using the numerical method of lines from the group of finite difference methods in detail described by E. H. Twizel [4]. This type of method can solve the second order nonlinear partial differential equation like nonlinear Schrödinger equation as a very useful equation for simulating the optical pulses propagation in fibers.

2. Instability analysis

Instability investigation in this case has been made with the SGP propagated in optical fibers. The start point for the simulation of this kind of problem is the simplified propagation equation [5], by ignoring the loss

$$i\frac{\partial A}{\partial z} = \frac{\beta_2}{2}\frac{\partial^2 A}{\partial t^2} - \gamma |A|^2 A,$$
(1)

where A(z,t) represents the amplitude of the field envelope, β_2 is the group velocity parameter responsible for dispersion, *t* denotes time and the nonlinear parameter γ is responsible for the self-phase modulation (SPM). The curtail is the SPM induced chirp in this case for a input Gaussian pulse in the from

$$\delta\omega(t) = \frac{2m}{t_0} \frac{L_{eff}}{L_{Nl}} \left(\frac{t}{t_0}\right)^{2m-1} \exp\left(-\left(\frac{t}{t_0}\right)^{2m}\right). \tag{2}$$

For the Gaussian pulse m=1 and m>1 for SGP, t_0 denotes the input pulse width and the L_{eff} and L_{NL} represent the effective and nonlinear length [6]. To achieve the modulation instability we do not consider the losses that mean the L_{eff} is not important for us. It is more useful to calculate the dispersion length L_D and nonlinear length L_{NL} as

$$L_{D} = \frac{t_{0}^{2}}{|\beta_{2}|} \qquad \qquad L_{NL} = \frac{1}{\gamma P_{0}}, \qquad (3)$$

where P_0 represents the power of the input pulse. Due to increasing the power or nonlinear coefficient we can change the SPM induced chirp, which can affects the stability of the propagated SGP pulse. In the last part of this article we focused also on the case of input soliton pulse and its instability.

3. Results

The main goal of the simulations that has been made was to find the parameters, especially input power P_0 and the nonlinear parameter γ by which we can observe the modulation instability of the propagated SGP. In Fig. 1 we can observe the shape of the propagated SGP with parameters described in detail below the figure. The crucial parameters for observation of the instability are the input power and the nonlinear parameter, which has a direct impact on the nonlinear length. Also we must note that we work in the anomalous dispersion regime. By changing this parameter and the nonlinear length we can directly influence the impact of the SPM induced chirp. We have increased these two parameters until we have observed the train of pulses in the middle. The initial decrease of the pulse intensity is due to the chirp, also this chirp is responsible for the pulse energy distortion.



Fig.1: The GP instability achieved by parameters m=2, $t_0=50$ ps, $P_0=100$ mW, $\beta_2=-1$ ps/nmkm and the nonlinear parameter $\gamma=0.005$ W⁻¹/m

Fig. 2 is used for the better observation of the energy distribution caused by the SPM induced chirp. These two figures described the instability and the energy distribution in case of the input super-Gaussian pulse. There are many articles describing soliton pulse modulation instability like in [7, 8, 9], but on the other hand we do not find many articles focused on the investigation of the modulation instability in case of SGP. In the next part of the article we also have focused on the most discussed case of the modulation instability in the case of soliton pulses. In Fig. 3 we can observe the modulation instability, that is responsible for the generation of sideband peaks, in the case of the input second order soliton pulse N=2 is calculated in this case using formula

$$\beta_2 = -t_0^2 / L_D.$$
 (4)

In the case of soliton pulse we nonlinear parameter in order of tenths of watts like in the case of the SGP because the balance between the dispersion and the nonlinear effects are much stronger compared with the SGP case. For the simulation of both pulse modulation instabilities the already mentioned numerical method of lines was used. This numerical method is very powerful in the case of solving the NLS equation for one channel transmission system.



Fig.2: The energy distribution for SGP caused by the SPM induced chirp

Fig. 4 is included for a better observation of the energy distribution in the case of soliton input pulse. Also in the case of soliton propagation to reach the modulation instability we must work in the anomalous dispersion regime. That can be very hard, because of finding a good interplay between the dispersion effects and nonlinear effects in the case of soliton generation. Dispersion length is set by the use of equation (3), which means the dispersion effects depend also on the input pulse width and on the group velocity dispersion through the term β_2 . Maybe in the soliton propagation regime the impact of SPM induced chirp can in the special case lead to a supercontinuum generation. For the future work it can be very interesting to use these results and the method to improve the case of generating a supercontinuum in the case of soliton propagated pulses and investigate if the supercontinuum can by generated due to the modulation instability.



Fig.3: The modulation instability in the case of the second order soliton pulse with the input parameters $t_0=50$ ps, $P_0=80$ mW and the nonlinear parameter $\gamma=0.3$ W⁻¹/m



Fig.4: The energy distribution for the second order soliton pulse and the spectrum broadening due to the SPM induced chirp

4. Conclusion

In this article we have used a numerical study to demonstrate the modulation instability in the case of SGP and also in the case of soliton input pulse. In the both cases the induced SPM chirp has the most impact on the modulation instability. The value of this chirp is largely dependent on the nonlinear parameter due to his direct impact on the nonlinear length. We have demonstrated in the simulations that the instability can by observed also in the case of the input super-Gaussian pulses not only in soliton case.

Acknowledgement

This work was partly supported by the Slovak Grant Agency under the project No. 1/1271/12 and the Slovak Research and Development Agency under the project No. APVV-0096-11.

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