# THEORY OF THE OPTICAL RADIATION SCATTERING IN MEDIA WITH PERIODICALLY CHANGING PERMITTIVITY IN BRAGG REGIME

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## 1. Introduction

When the acoustic ultrasonic wave propagates in an optically transparent medium (water, $LiNbO_3$, GaP, $TeO_2$ etc.), it produces a periodic modulation of the index of refraction (or relative permittivity) via the elasto-optical effect. This provides a relative slowly moving phase grating which can generate scatter (or diffract?) portions of an incident optical radiation into one or more directions. This phenomenon, known as the acoustooptic scattering, has led to a variety of optical devices that perform spatial, temporal, and spectral modulations of optical radiation.

The acoustooptic scattering is usually explained as a collision of photons and phonons [1]. For these quasi-particles to have a well defined momenta and energies, we must assume, that we have interaction of mono-chromatic plane waves of optical radiation and sound. The width of the transducer $L$ must be assumed to be sufficient-ly wide in order to produce plane wave fronts at a single frequency (Fig. 1). In the process of collisions two conservations laws have to be obeyed, namely, the conservation of energy and momentum. In the case where $L$ is sufficiently short, we have a first form of a scattering, called Raman-Nath scattering. The condition $L \ll \Lambda^2/\lambda$ defines therefore the *Raman-Nath scattering regime* [2]. In this regime many scattering angles may exist because various directions of plane waves of sound are provided from a small-aperture transducer. If $L \gg \Lambda^2/\lambda$ ($\Lambda$ is the sound wavelength, $\lambda$ is the optical radiation wavelength in the acoustic medium), only scaterring angles $\Phi_0$ and $\Phi_{-1}$ are observed. In this case the acoustooptic device operates in the so called *Bragg regime* since this is similar to what happens in Bragg scattering of X-rays or electrons on a crystal lattice. The „grating constant“ in this case is equal to sound wavelength and the required angle of incidence $\Phi_{inc}$ is given by well-known Bragg condition $2\Lambda\sin\Phi_{inc} = \lambda$ [1, page 557].

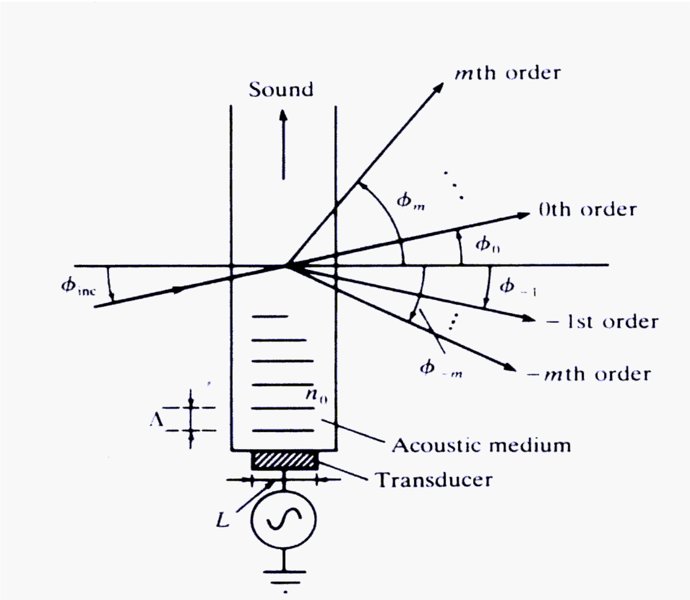


Fig. 1 *Basic acoustoptic device* (*taken from* [1] )

In our previous work [3] we have presented a model of the optical radiation intensity distribution in the Raman-Nath acoustoptic scattering regime. In this work a possible semi-classical model of the intensity of scattered optical radiation in the Bragg regime is proposed and briefly presented.

## 2. Theory

The Bragg condition $2\Lambda \sin\Phi_{inc} = \lambda$ does not specifically describe the real intensity distribution when the optical radiation passes through the transparent sample in which the plane ultrasonic acoustic wave propagates. It defines only the angle of incidence needed for creation of intensity scattering maxima appearing at two angles $\Phi_{-1}$ and $\Phi_0$. The acoustoptic cell shown in Fig. 1 may be thought [3] to act as a thin phase grating with an effective grating line separation equal to the wavelength $\Lambda$ of the sound. It is well known that a phase grating split incident optical radiation in general into many various orders of scattering maxima, not only into two directions. The main directions of the scattered (or diffracted) radiation maxima inside the sound cell are governed by the „grating equation“ with non-zero angle of incidence, typical for diffraction many-slit optical grating

$$\sin\Phi_m = \sin\Phi_{inc} + m\frac{\lambda}{\Lambda} = \sin\Phi_{inc} + m\frac{k}{K}; \quad m = 0; \pm 1, \pm 2,... \tag{1}$$

Hence, the angle between neighbouring orders in Fig. 1 is equal to $\lambda/\Lambda$ inside the cell. (Outside the cell this angles are increased through refraction on the rear interface; $k$ and $K$ are propagation constants of the radiation and sound waves inside the cell.) The energy conservation law leads to the equation

$$\omega_m = \omega \pm m\Omega \tag{2}$$

with $\omega_m$ being the frequency of the $m$-th order of scattered optical radiation, $\omega$ is the frequency of incident radiation and $\Omega$ is the frequency of sound wave (sound velocity is about 1-7 km/s). The (very small) changes of the frequencies are due to (in general multiple) photon-phonon interactions (connected with emission or absorption of phonons). We will assume that $x$ axis is in the sound direction and $z$ axis lies in the direction from left to right in Fig. 1. The next analysis is in some aspects similar to work [4]. We assume an isotropic inhomogeneous non-magnetic non-conductive medium without birefringence.

The interaction between the electromagnetic wave $\boldsymbol{E}(x, z, t)$ (polarized along the $y$ axis) and longitudinal sound wave field $S(z, x, t)$ can be described by Maxwell´s equations. The time varying permittivity can be written [3] as

$$\varepsilon(x,z,t) = \varepsilon + \varepsilon_1(x,z,t); \quad \varepsilon_1(x,z,t) = \varepsilon CS(x,z,t)) \tag{3}$$

e. g. the time dependent part of permittivity $\varepsilon_1$ is proportional to the sound field amplitude $S$ and $C$ is material constant of medium. From Maxwell´s equation (assuming $\boldsymbol{E}.grad\ \varepsilon(x,z,t) = 0$) we can obtain the wave equation in the form

$$\nabla^2 E(x,z,t) = \mu_0 \frac{\partial^2}{\partial t^2}[\varepsilon(x,z,t)E(x,z,t)] \tag{4}$$

Because the time variation of $\varepsilon(x,z,t)$ is much slower than that of $E$, we will only retain a term which does not contain the time differentiations of $\varepsilon(x,z,t)$:

$$\nabla^2 E(x,z,t) - \mu_0 \varepsilon \frac{\partial^2 E(x,z,t))}{\partial t^2} = \mu_0 \varepsilon_1(x,z,t) \frac{\partial^2 E(x,z,t))}{\partial t^2} \tag{5}$$

It is a wave equation often used to investigate strong interactions in acoustoptics. We will now introduce harmonic variations in the optical radiation and sound as complex functions in the forms

$$E(x,z,t) = \frac{1}{2}\sum_{m=-\infty}^{\infty} E_m(x,z)\exp[i(\omega + m\Omega)t] + c.c.\ ; \quad \frac{\varepsilon_1(x,z,t)}{\varepsilon} = \frac{1}{2} CS(x,z)\exp(i\Omega t) + c.c. \tag{6}$$

where c.c. denotes complex conjugate. We assume the frequency mixing of fotons and acoustic phonons in the time-dependent phase of $E$ according to equation (2). Substituing (6) to (5) and assuming $\Omega \ll \omega$ we obtain the infinite coupled-wave system

$$\nabla^2 E_m(x,z) + k^2 E_m(x,z) + \frac{1}{2}k^2 CS(x,z)E_{m-1}(x,z) + \frac{1}{2}k^2 S^*(x,z)E_{m+1}(x,z) = 0;\ k = \omega\sqrt{\mu_0 \varepsilon} \tag{7}$$

where $k$ is propagation constant of electromagnetic (optical) radiation and the asterisk denotes complex conjugate. The quantity $E_m$ is the complex spatial part of the $m$th order radiation wave function at frequency $\omega + m\Omega = \omega_m$.

We will now consider a uniform sound wave of the width of transducer $L$ propagating along $x$ axis:

$$S(x,z) = S(x) = S_0 \exp(-iKx); \quad K = \frac{2\pi}{\Lambda} \tag{8}$$

We expect that the spatial part of wave function $E_m$ can be written as

$$E_m(x,z) = E_{0m}(x,z)\exp(-ikz\cos\Phi_m - ikx\sin\Phi_m) \tag{9}$$

with the choice of angle $\Phi_m$ according to eq. (2). Substituing (8), (9) and (2) into (7) we obtain

$$\begin{aligned} &\frac{\partial^2 E_{0m}}{\partial x^2} - 2ik\sin\Phi_m \frac{\partial E_{0m}}{\partial x} - 2ik\cos\Phi_m \frac{\partial E_{0m}}{\partial z} + \frac{1}{2}k^2 CS_0^* E_{0m+1}\exp[-ikz(\cos\Phi_{m+1} - \cos\Phi_m)] + \\ &\quad + \frac{1}{2}k^2 CS_0 E_{0m-1}\exp[-ikz(\cos\Phi_{m-1} - \cos\Phi_m)] = 0 \end{aligned} \tag{10}$$

Here we have assumed that within a wavelength $\lambda$ the quantity $E_{0m}$ and its first derivative does not change appreciably with $z$ and hence the term $\partial^2 E_{0m}/\partial z^2$ can be neglected. The last two terms in (10) represent the interactions between adjacent orders of scattered optical radiation with the sound waves. The first two terms on the left-hand-side are responsible for propagational diffraction and the third term represents the effect of the $m$th order of radiation travelling in direction slightly different from $z$. If the width $L$ is not large the propagational diffraction can be neglected. Because the angles $\Phi_m$ are very small (if $\Phi_{inc}$ is very small or zero) we can assume that $E_{0m}$ depends only from $z$. The equation (10) then becomes

$$\begin{aligned} \frac{\partial E_{0m}}{\partial z} = &-\frac{1}{4ik\cos\Phi_m} k^2 CS_0^* E_{0m+1}\exp[-ikz(\cos\Phi_{m+1} - \cos\Phi_m)] - \\ &-\frac{1}{4ik\cos\Phi_m} k^2 CS_0 E_{0m-1}\exp[-ikz(\cos\Phi_{m-1} - \cos\Phi_m)] \end{aligned} \tag{11}$$

which can be solved with the boundary condition ($\delta_{m0}$ is the Kronecker delta function):

$$E_{0m} = E_{0inc}\delta_{m0} \tag{12}$$

The physical interpretation of (11) is that there exists a mutual coupling between neighboring orders of scattered radiation. But the phase of contributions varies with $z$ and so the terms in exponential functions represent the lack of phase synchronism in this coupling process.

Unlike of the Raman-Nath regime in the Bragg scattering regime the transducer thickness is sufficiently large and the phase synchronism between neighboring orders of wave functions can occur only for $m$ = 0 and -1. Then the exponential factors in (11) are equal to zero. This implies the condition $\Phi_{-1} = -\Phi_0 = \Phi_{inc}$ i. e. the first scattering maximum occurs in the direction of incident beam and the second maximum in direction $-\Phi_{inc}$ only. It is typical for the Bragg scattering regime. The coupling equations between two modes of the wave functions are according to (11) and (12)

$$\frac{\partial E_{-1}}{\partial z} = -\frac{1}{4i\cos\Phi_{inc}} k_0 C S_0^{*} E_0 \quad ; \quad \frac{\partial E_0}{\partial z} = -\frac{1}{4i\cos\Phi_{inc}} k_0 C S_0 E_{-1} \tag{13}$$

The solutions for $z$ = 0 to $L$ and for small but *non-zero* $\Phi_{inc}$ read for wave functions of the two scattering maxima

$$E_0 = E_{inc}\cos\left(\frac{k_0 C|S_0|L}{4}\right) \qquad E_{-1} = -i\frac{S^{*}}{|S|}E_{inc}\sin\left(\frac{k_0 C|S_0|L}{4}\right) \tag{14}$$

These equations represent the expressions for the optical radiation scattered on acoustic waves in the Bragg regime.

## 3. Conclusion

The observable quantity, e. g. mean time value of the intensity of scattered optical radiation is equal to $(1/2)\varepsilon_0 c E_0^2$ or $(1/2)\varepsilon_0 c E_{-1}^2$ . The presented model describes in such a way the layout of intensities of two scattering maxima observable in standard acoustooptical device operating in Bragg regime.

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