FORWARD TRANSFORMATION FOR HIGH RESOLUTION EDDY CURRENT TOMOGRAPHY USING WHITNEY ELEMENTS

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1. Introduction

Tomographic methods are intensively developed field of non-destructive testing [1]. The main advantage of this type of NDT method is 3D information concerning the shape of the discontinuities in investigated material. On the other hand, the most common tomography method utilizing X-rays creates the significant risks typical to X-ray technique. As a result, X-ray tomography is difficult to use in industry [2].

Introduced to the industry in 2007 [3], the eddy current tomography is safe and cost effective. However, in opposite to X-ray tomography, eddy current tomography requires sophisticated and time consuming inverse transformation creating 2D or 3D view of discontinuities. For this reason solutions presented previously are focused on 2D inverse transformation and exhibit limited resolution [4].

For eddy current tomography, the forward tomographic transformation is most important, which is the base of inverse transformation. This paper presents the novel, fast and cost-effective solution utilizing Whitney elements method [5] for such forward transformation. As a result, new possibilities of development in the area of high resolution 3D eddy current tomography are created.

2. Reference tomographic measuring stand

Figure 1a presents the testing element for eddy-current tomography experiments. This element consist a cylinder with prism-shaped inclusion as it is presented in figure 1a. Cylinder has 30 mm of diameter and 120 mm height, whereas all prisms created 13 mm deep and 12mm wide inclusion. During the investigation constructional steel cylinder with copper prism was used.

The proposed mechanical system for eddy current tomography experimental setup is presented in figure 1b. The setup is based on two coils (2): exciting coil and detecting coil. Tested object (1) is transported between these coils. Linear (3) and rotary (4) actuator provide appropriate movement of the object. As a result tomographic measurements may be done with linear movement resolution of up to 0,1 mm, whereas rotation resolution during the measurements is up to 1^{o}.

Due to the fact, that developed eddy currents tomographic system is digitally controlled, resolution of measurements may be increased or decreased appropriately to required resolution as well as capacities of data processing system.

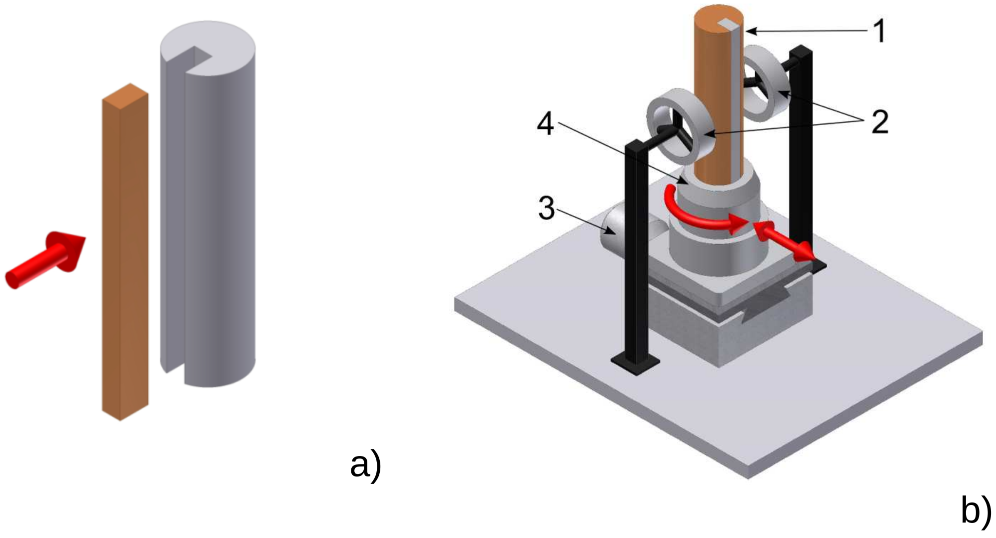

Fig.1: *Reference tomographic measuring stand: a) model element for eddy current tomography experiments - steel rod with copper inclusion, b) mechanical setup of eddy current tomography device: element under investigation (1), driving and sensing coils (2), linear (3) and rotational (4) drivers*

3. Modelling

Forward transformations in eddy current tomography are based on the Maxwell equations. However, due to the fact, that for eddy current tomography, the magnetic field frequency is limited up to about 2 kHz, the Maxwell equations may be presented in simplified, eddy current oriented form [6]:

$$\frac{\partial \boldsymbol{B}}{\partial t} + \nabla \times \boldsymbol{E} = 0 \tag{1}$$

$$\nabla . \boldsymbol{B} = 0 \tag{2}$$

$$\nabla \times \boldsymbol{H} = \boldsymbol{J} \tag{3}$$

$$\boldsymbol{B} = \mu \boldsymbol{H} \tag{4}$$

$$\boldsymbol{J} = \sigma \boldsymbol{E} \tag{5}$$

where $\boldsymbol{B}$ is magnetic flux density, $\boldsymbol{J}$ is current density, μ is the magnetic permeability, and σ is the electric conductivity. If we denote magnetic vector potential as $\boldsymbol{A}$, the equations (1-5) lead to the following equation [7]:

$$\nabla \times (\frac{1}{\mu} \nabla \times \boldsymbol{A}) + i\omega\sigma \boldsymbol{A} = \boldsymbol{J} \tag{6}$$

Moreover, we can define scalar potential V as follow:

$$\boldsymbol{E} + \frac{\partial \boldsymbol{A}}{\partial t} = -\nabla V \tag{7}$$

which will lead to eddy current oriented Maxwell equations in the A-V form. The approximation of the associated vector potential variable $\boldsymbol{A}$ can be done by the use of the lowest-order edge element discretization, while the Lagrange interpolation may be applied to compute the scalar potential V. In addition to performing the computations in the time domain, the analogous version of the equations may also be solved in the frequency domain considering weak formulations. Finally, nodal approximations of derived fields $\boldsymbol{B}$ and $\boldsymbol{J}$ may be calculated, after the two potentials have been obtained [8].

Such methodology of solving the eddy current oriented Maxwell equations is implemented in open-source Elmer software [9]. However, Elmer is mainly oriented on solving partial differential equation. For this reason, to perform forward transformation for eddy current tomography all process was controlled by the Octave software. Then, under the control of Octave [10], tetrahedral network imaging elements was generated using Netgen software [11]. Finally, eddy current oriented Maxwell partial differential equations were solved by Elmer. Results of these calculations were exported to Octave for further processing and analyses.

4. Results of forward transformation

Figure 2 presents the results of forward tomographic transformation. In figure 2a the distribution of flux density B in the direction of driving coil axis can be observed, whereas in figure 2b, the eddy current J_c vectors are given.

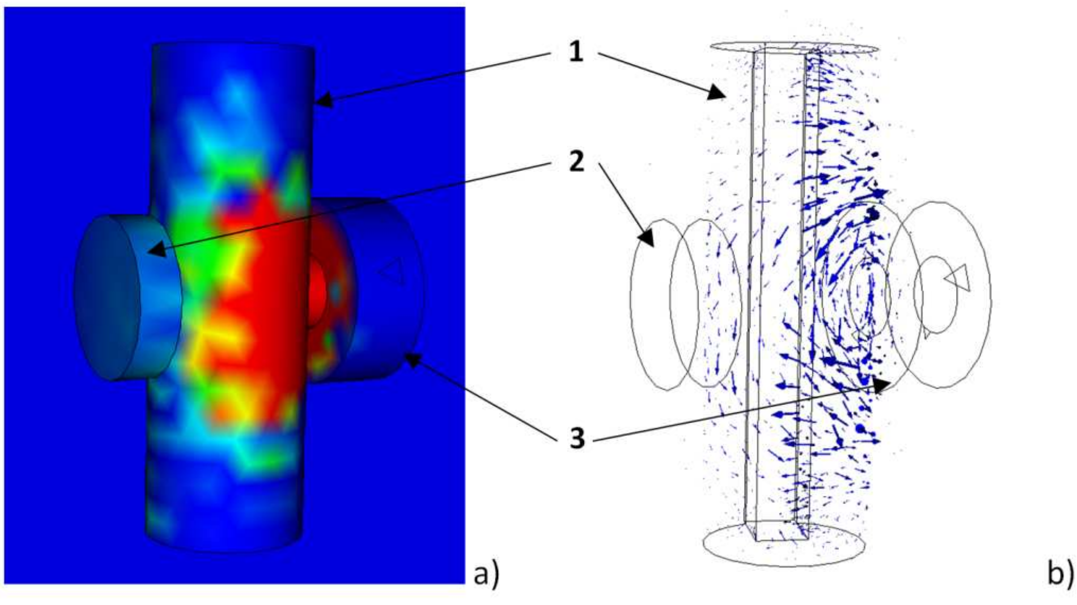

Fig. 2: *Model of magnetizing field distribution and eddy currents in the modeled object: 1 – cylindrical test object, 2 – measuring coil, 3 – exciting coil.*

Calculations were carried out using one core of 8-cores Intel i7-4770 CPU operating at 3.5 GHz. PC was controlled by Scientific Linux 6.5. Calculations covered 72495 nodes and 429354 elements. Calculations were carried out in 120 s (together with mesh generation), and used about 6 GB of RAM memory.

5. Conclusion

Presented results confirm, that eddy current oriented Maxwell equations in A-V form may be successfully solved in acceptable time even for high density meshes. As a result the effective method for forward tomographic transformation was developed. This method creates the new possibilities for development of eddy current NDT imaging system with the never previously presented resolution.

References:

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