SENSITIVITY ANALYSIS IN NUCLEAR POWER ENGINEERING

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1. Introduction

Within the reactor analysis and design calculation, sensitivity analysis offers to a nuclear engineer unique insight into investigated system. Estimation of the change of the system response, due to change in some input parameter, can identify important processes and evaluate the influence of variation in this parameter. Defining the critical eigenvalue or reactivity as our response, allows us to use the perturbation theory to determine the sensitivity coefficients of the system. Main constraint in this methodology is the assumption that a considered perturbation is small enough to cause a change in the neutron flux. Otherwise it is not necessary to perform new calculation for the perturbed system, which decreases the computational time requirements. The sensitivity coefficients can be further used for calculation of k_{eff} or other response uncertainty coming from cross section data; or sensitivity coefficients of reactivity response by using critical eigenvalue sensitivities for perturbed and unperturbed state. The responses which can be addressed with perturbation theory are; multiplication factor, peak to average power, control rod worth, breeding ratio, delay neutron fraction and moderator void coefficient [1].

2. Theory

Derivation of sensitivity for listed responses is based on Standard or Generalized Perturbation Theory (GPT). Due to fact that Standard Perturbation Theory (SPT) is a special case of GPT and multiplication factor is the most evaluated parameter, important steps of derivation of k_{eff} sensitivity are presented below.

The reactor multiplication factor can be defined as the fundamental eigenvalue in the neutron balance equation for multiplying system

$$\boldsymbol{L\Phi}(\boldsymbol{x}) - \boldsymbol{\lambda P\Phi}(\boldsymbol{x}) = \boldsymbol{0}, \tag{1}$$

where x symbolically represents all independent variables such as the neutron's space, energy and direction coordinates. L and P are net loss and production Boltzman operators respectively and λ is lambda mode eigenvalue where $\lambda = 1/k_{eff}$. A change in some input parameter α appearing in the L and P operators will perturb the neutron balance which also cause change in the eigenvalue. By introducing the perturbed parameters to Eq. (1), multiplying by non-zero weighting function $w(x)$, integrating over phase-space and solving it for the change in the eigenvalue we get

$$\boldsymbol{\Delta\lambda} = \frac{\langle \boldsymbol{w}(\boldsymbol{\Delta L} - \boldsymbol{\lambda\Delta P})\boldsymbol{\Phi}\rangle + \langle \boldsymbol{w}(\boldsymbol{L} - \boldsymbol{\lambda P})\boldsymbol{\Delta\Phi}\rangle}{\langle \boldsymbol{wP\Phi}\rangle} \tag{2}$$

where all products of perturbations are neglected and angle brackets ⟨ ⟩ represent integration over phase-space (volume, energy and direction). Eq. (2) represents the first-order estimate for the eigenvalue perturbation. If $w(x)$ is set equal to λ-mode adjoint flux Φ^*, the second term in the numerator of Eq. (2) vanish. If the weighting function in Eq. (2) is selected to be solution

of the perturbed adjoint equation, it is possible to obtain the exact value for a change in the multiplication factor and for any size of perturbation. This principle can be applied also in the direct perturbation method.

It is common in perturbation analysis to deal with relative changes and we are looking for expression for k_{eff}, so that Eq. (2) can be written in the form

$$S_{k,\alpha} = \frac{\alpha}{k}\frac{\Delta k}{\Delta \alpha} \cong \frac{\Delta \alpha}{\alpha}\frac{\langle \Phi^*\left(\frac{1}{k}\frac{\partial P}{\partial \alpha}\alpha - \frac{\partial L}{\partial \alpha}\alpha\right)\Phi\rangle}{\frac{1}{k}\langle \Phi^* P \Phi\rangle}, \quad (3)$$

where $S_{k,\alpha}$ is the sensitivity coefficient of k_{eff} with respect to α.

Based on Equivalent GPT, sensitivity coefficients of eigenvalue determined for two states of one system can be used to calculate sensitivity coefficients of a reactivity, describing this difference as a sodium coefficient, rod worth or Doppler effect. Based on [10, 11] the sensitivity coefficient of the reactivity to parameter α is defined as

$$S_{\rho,\alpha} = \frac{\Delta \rho}{\Delta \alpha}\frac{\alpha}{\rho} = \frac{1}{\rho}\left[\frac{S_{k2}}{k_2} - \frac{S_{k1}}{k_1}\right], \quad (4)$$

where ρ is the reactivity change between two pre-defined states expressed, and $S_{k1,2}$ is sensitivity coefficient of k_{eff} to parameter α defined by Eq. (3). In addition sensitivity coefficients are often used to relate the cross section uncertainties to the uncertainty in the response, k_{eff} or reactivity [1]. The uncertainty in the response *R* is then given approximately by

$$\sigma_R^2 = S_{R,\alpha} C_\alpha S_{R,\alpha}^T, \quad (5)$$

where σ_R^2 is the variance of the response *R* and C_α is a covariance matrix of parameter α.

3. Calculation methods

Theory presented in the previous section was applied within a development of the sensitivity analysis code APSTRACT [2]. Calculation provided by this code is based on 150 group MATXS and ISOTX format data library prepared from ENDF/B-VII nuclear data library [3]. Flux solver TWODANT, transport code included in the modular DANTSYS [4], was chosen to determine angular forward and adjoint flux. As an alternative approach, 238 group sensitivity data sets were independently generated by SCALE6.1.3 system where stochastic Monte Carlo KENO-VI module was used as a neutron solver and sensitivity coefficients were calculated by TSUNAMI module [5]. The sensitivity coefficients calculated by TSUNAMI module had to be collapsed to 33 group structure. For this case, auxiliary utility for group collapsing was developed.

In order to make demonstration of the sensitivities code capability, seven modified benchmark problems defined by Subgroup 33 were investigated, namely: Joyo, ZPPR9, ZPR6-7 (standard configuration) and ZPR6-7 (High ^{240}Pu content) Jezebel (^{239}Pu and ^{240}Pu configuration) [6, 7].

4. Discussion and results

The comparisons of sensitivities for chosen reactions and for most important isotopes are presented below. Tab. 1 show sensitivity coefficients determined by SPT technique and direct perturbation calculation (DP) where the 1% change of cross section data was applied.

Tab. 1. *Integral sensitivity coefficients in % of k_{eff} calculated by direct perturbation (DP) and standard perturbation theory (SPT)*

Benchmark	Jezebel Pu239		ZPPR9		Joyo	
Isotope	^{239}Pu		^{238}U		^{235}U	
Type	DP	SPT	DP	SPT	DP	SPT
fission	7.21E-01	7.28E-01	9.12E-02	9.14E-02	2.76E-01	2.78E-01

nubar	9.57E-01	9.66E-01	1.47E-01	1.48E-01	4.60E-01	4.62E-01
capture	-7.40E-03	-7.54E-03	-2.69E-01	-2.69E-01	-4.78E-02	-4.77E-02
elastic	6.50E-02	6.42E-02	2.66E-02	2.67E-02	4.79E-03	4.85E-03
inelastic	4.00E-02	3.99E-02	-6.04E-02	-6.05E-02	6.91E-04	7.16E-04

The sensitivity coefficients of individual benchmark cases are in a good accordance with those achieved from direct perturbation as is presented in Tab. 1. Both methods lead to the same results, which is consistent with the theory. In all figures, the abbreviation "Aps33" denotes results calculated by APSTARCT, and "SCALE" represents results calculated by TSUNAMI.

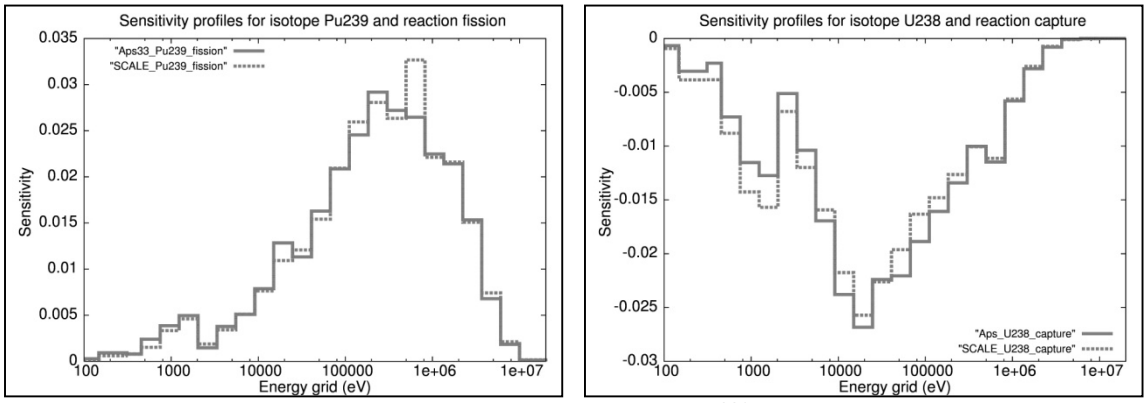

Fig.1: *Sensitivity profile of k_{eff} for Joyo benchmark and ^{239}Pu fission in left and sensitivity profile of k_{eff} for ZPR6-7 benchmark and ^{238}U capture in right.*

Sensitivity profiles of the representative isotopes and reactions for fast ractors are presented in Fig. 1. Sensitivty of k_{eff} to fission on ^{239}Pu identifies energy range where the fission occured with highest probability as well as the sensitivty of k_{eff} to ^{238}U capture cross section, the most important resonances.

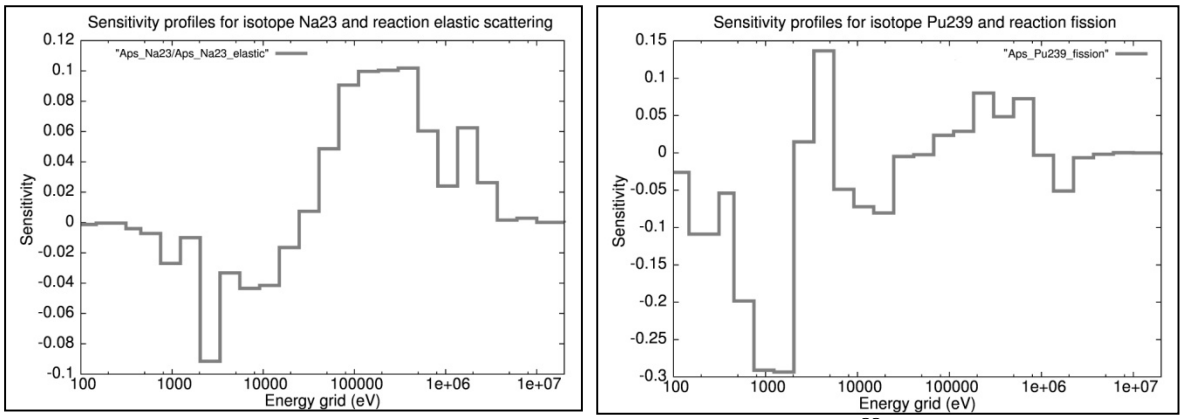

Fig. 2: *Sensitivity profiles of reactivity for ZPPR9 benchmark and ^{23}Na elastic scattering in left and ^{239}Pu fission in right.*

Fig. 2 demonstrates sensitivity of the reactivity to cross section data. As you can see, the absolute value of the sensitivity coefficients is higher compare with sensitivity coefficients of k_{eff}. Within this method two sensitivity profiles of k_{eff} are compared and both are affected by the same cross section data, therefore final sensitivity or effect is stronger which has also impact to the final value of uncertainty. In this case the sodium void effect was investigated.

Tab. 2. *Total uncertainty of k_{eff} coming from cross section data for investigated benchmarks*

Code	Flattop	Jez_Pu^{239}	Jez_Pu^{240}	Joyo	ZPPR9	ZPR6-7HPu	ZPR6-7 STD
APS	0.7104	0.5977	0.5029	0.6071	1.1698	1.0178	1.0717
SCALE	1.2206	1.3712	1.2099	1.2870	1.4574	1.2829	1.2920

Application of the sensitivity coefficients to evaluate k_{eff} uncertainty is simple, which is demonstrated by Eq. (5), but the correct interpretation of these uncertainties is quiet difficult because they are strongly dependent to the quality of covariance matrix. In Tab. 2 total uncertainties of k_{eff} for each investigated benchmark are presented. Calculation was based on two different covariance data files. For APSTRACT case, the covariance data were prepared by standard NJOY procedure for fast reactor application, but only for limited number of isotopes, while in SCALE case the 44 group SCALE covariance library was used.

5. Conclusion

Sensitivity analysis methods offer to nuclear engineer variety of application. Easily can be identified important processes, reactions or isotopes through sensitivity coefficients, presented in Fig. 1. By the direct perturbation calculation method, evaluator can validate his results calculated by SPT, which was demonstrated in Tab.1. The exact perturbation is able to decomposed a reactivity effects to contributed processes. By using two sets of k_{eff} sensitivities, describing some reactivity change, sensitivity of this reactivity to cross section data can be calculated. Finally, all sensitivity profiles are suitable for evaluation of the corresponding response uncertainty coming from the cross section data. But final uncertainties dependent mainly on the covariance data because sensitivities in this process serve as a weighting function.

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