EXTRACTION OF OPTICAL FIBER LOCAL BIREFRINGENCE PARAMETERS FROM THE DATA MEASURED BY PO-OTDR METHOD

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1. Introduction

Except of the contemporary broad application of optical fibres in modern transmission telecommunication and information systems they are now also frequently used for wide sensory applications especially in fibre optic sensors with distributed parameters (FOSDP). These sensors are based on the local interaction of optical fibre with various external physical quantities like mechanical stress, tension, friction, temperature, electric and magnetic field intensity and so on. These quantities induce the local changes of the index of refraction in the fibre and subsequently the changes of the optical radiation parameters in the fibre. It concerns not only the amplitudes of the optical waves (modes) but also their polarization properties. The modes polarization is very sensitive to the above mentioned external factors. One of the most significant method that makes possible to measure local polarization parameters of the propagating optical wave in the optical fibre is Polarization Optical Time-Domain Reflectometry (PO-OTDR) that is one of the several existing variations of the original OTDR that was discovered and realized in 1976 by Barnoski and Jensen [1]. The basic idea of the OTDR consists in launching a short impulse of optical radiation into the test fibre. During the propagation along the fibre a part of impulse energy is scattered isotropically by the mechanism of Rayleigh scattering on the microscopic fluctuations of the index of refraction what reflects also in attenuation of the impulse [2]. A part of the scattered energy proportional to the numerical aperture and other parameters of the fibre is captured by the fibre core and directed to the input end of the fibre where it is detected and measured.

PO-OTDR is based on the same principle as OTDR. Only the polarization controller is added before input end of the fibre enabling to set the input polarization and a polarization analyser at the end of the fibre making possible to measure the Stokes polarization parameters of the backscattered radiation [3]. From the practical point of view it is significant that OTDR and also PO-OTDR measurement can be done with the access to only one input end of the fibre.

The main task and problem of the PO-OTDR is the transformation of measured polarization parameters of the back-scattered light into the distribution of the full local polarization parameters along the fibre and consequently into the local physical parameters of the fibre determining the total local fibre birefringence [4].

At the beginning of this paper we describe analytically in terms of Stokes vectors and Mueller matrices the space evolution of the polarization of the optical radiation propagating in the *homogeneous single mode birefringent optical fibre*. The main focus is on the *analysis of the matrix based approach to the extraction of the local optical fibre birefringence parameters* from the measured data by the PO-OTDR.

2. Birefringence description in homogeneous optical fibre

To be able to analyse the axial distribution of the light polarization along a non-homogeneous optical fibre it is at first necessary to describe this problem in a homogeneous fibre.

As it is generally known in an ideally homogeneous and isotropic single-mode optical fibre two independent degenerated orthogonally polarized modes can propagate. The corresponding propagation constants of these modes are equal and the modes do not mutually exchange the energy. There is no interaction between them. However due to the index of refraction microscopic fluctuations and accidental changes of fibre geometrical parameters and also due to the influence of some external factors (temperature, tension,…) the modes degeneration is lost. In such a way the propagation constants of the original non-interacting modes become functions of the position on the fibre and mutual coupling between modes takes place.

Let us define the complex amplitudes $a_x(z)$, $a_y(z)$ of both modes (harmonic plane waves with the time dependence $e^{+j\omega t}$) as a function of the propagation distance "z" along the z axis of the orthogonal coordinate system which is identical with the fibre axis. Then using the weak coupling approximation [5] one can write for the resulting complex amplitude of the total electric field the following relation

$$\vec{\mathbf{E}}(x,y,z) = a_x(z)\vec{\mathbf{E}}_x(x,y) + a_y(z)\vec{\mathbf{E}}_y(x,y) \tag{1}$$

and the influence of mutual coupling between modes can be described generally by two first order differential equations of the following form

$$\frac{d}{dz}\begin{bmatrix} a_x(z) \\ a_y(z) \end{bmatrix} = -j\begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix}\begin{bmatrix} a_x(z) \\ a_y(z) \end{bmatrix} \tag{2}$$

where k_{ij} are mode coupling coefficients and d/dz represents the z derivative. Except of coupling coefficients k_{ij} and the coordinate z the solution of these equations depends also on two initial conditions $a_x(0)$, $a_y(0)$ that define the input SP.

$$\begin{bmatrix} a_x \\ a_y \end{bmatrix} = f\left(a_x(0), a_y(0), z, k_{ij}\right) \tag{3}$$

The defined formulation is not quite suitable mainly due to the necessity of rather demanding measurement of complex amplitudes at optical frequencies. This can be avoided by the transformation of that description system (Jones matrix) into one based on 4 Stokes parameters $S_i(z)$ (Mueller matrix) [6] defined by the use of the above introduced complex amplitudes in the following way

$$\begin{aligned} S_0(z) &= a_x(z)a^*_x(z) + a_y(z)a^*_y(z) \\ S_1(z) &= a_x(z)a^*_x(z) - a_y(z)a^*_y(z) \\ S_2(z) &= a_x(z)a^*_y(z) + a_y(z)a^*_x(z) \\ S_3(z) &= j\left[a_x(z)a^*_y(z) - a^*_x(z)a_y(z)\right] \end{aligned} \tag{4}$$

Combining (4) and (2) and neglecting the optical loses ($dS_0/dz = 0$, S_0 = const.) it is possible to write the system of differential equations for the Stokes parameters in the form:

$$\begin{bmatrix} \frac{dS_1(z)}{dz} \\ \frac{dS_2(z)}{dz} \\ \frac{dS_3(z)}{dz} \end{bmatrix} = \begin{bmatrix} 0 & 2k_2 & 2k_1 \\ -2k_2 & 0 & \Delta \\ -2k_1 & -\Delta & 0 \end{bmatrix} . \begin{bmatrix} S_1(z) \\ S_2(z) \\ S_3(z) \end{bmatrix} \tag{5}$$

where $K = k_{12} = k^*_{21} = k_1 + jk_2$ a $\Delta = k_{22} - k_{11}$.

Without depolarization it holds $S^2_0 = S^2_1 + S^2_2 + S^2_3$ and the solution of (5) makes possible to get the expression describing the dependence of Stokes parameters on the distance z *along the homogeneous optical fibre*. If k_{ij} are supposed *not to depend on z coordinate* the system of equation (5) has the solution [7]

$$\boldsymbol{S}(z)=S_1(z)\boldsymbol{u}_1+S_2(z)\boldsymbol{u}_2+S_3(z)\boldsymbol{u}_3=[\boldsymbol{S}(0).\boldsymbol{S}_P].\boldsymbol{S}_P+[\boldsymbol{S}(0)-[(\boldsymbol{S}(0).\boldsymbol{S}_P)\boldsymbol{S}_P]].\cos[(\delta)z)] \pm [\boldsymbol{S}(0)x\boldsymbol{S}_P].\sin[(\delta)z], \tag{6}$$

where $\boldsymbol{S}(0)= S_1(0)\boldsymbol{u}_1+ S_2(0)\boldsymbol{u}_2+ S_3(0)\boldsymbol{u}_3$ and $\boldsymbol{S}_P=S_{1p}\boldsymbol{u}_1+ S_{2P}\boldsymbol{u}_2+S_{3P}\boldsymbol{u}_3$ are the Stokes vectors of the input polarization and one of the two vectors of principal state of polarization (PSP) respectively. PSP are those that do not change along the propagation. From (6) one can deduce that the output SP vector $\boldsymbol{S}(z)$ rotates around the PSP vector $\boldsymbol{S}_P$ under the constant angle tilted between the $\boldsymbol{S}(0)$ and $\boldsymbol{S}_P$.

The PSP Stokes vector components S_{iP} can be calculated as follows [8]:

$$S_{1P} = \pm\frac{\Delta}{\delta} \quad S_{2P} = \mp\frac{2k_1}{\delta} \quad S_{3P} = \pm\frac{2k_2}{\delta}\,, \quad \delta = \sqrt{\Delta^2 + 4|K|^2} \tag{7}$$

where δ is the absolute value of the fibre elliptic birefringence vector representing the difference between the propagation constants of the two PSP. The derived relations make possible to calculate the local SP distribution along the fibre or the dependence of the output SP as a function of coupling coefficients which can be influenced by various external physical magnitudes. If the fibre length z=*l* is sufficiently short and the external factor is constant along that short fibre it is possible to calculate the coupling coefficients using the measured Stokes vectors [*S(z=l)*] for a given [*S(z=0)*]. If the mechanism of transformation of an external physical quantity change into change of coupling coefficients is known it is possible to use the measured changes of corresponding Stokes vectors for the calculation of external quantity change. It is a crucial point of the application of the OF as OFSDP.

It is important to stress that the results achieved above are useable only if the input SP is given. Due to the fact that the PSP Stokes vector components (7) that depend on Δ and k_1 ,k_2 are generally not known it is not possible to calculate these parameters from one measurement of Stokes parameters. This problem can be solved by the introduction of the MM formalism relating the input SP [*S(0)*] with the output one [*S(z)*]. It can be expressed by the following linear matrix equation

$$[S(z)] = [M(z)].[S(0)] \;, \tag{9}$$

where

$$[M(z)] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & S_{1P}^2 + \left(1 - S_{1P}^2\right)\mathrm{C} & S_{1P}S_{2P}(1-\mathrm{C}) \pm S_{3P}\mathrm{S} & S_{1P}S_{3P}\left(1-\cos(\delta \mathrm{z})\right) \mp S_{2P}\mathrm{S} \\ 0 & S_{1P}S_{2P}\left(1-\cos(\delta \mathrm{z})\right) \mp S_{3P}\mathrm{S} & S_{2P}^2 + \left(1 - S_{1P}^2\right)\mathrm{C} & S_{2P}S_{3P}\left(1-\cos(\delta \mathrm{z})\right) \pm S_{1P}\mathrm{S} \\ 0 & S_{1P}S_{3P}\left(1-\cos(\delta \mathrm{z})\right) \pm S_{2P}S & S_{1P}S_{3P}\left(1-\cos(\delta \mathrm{z})\right) \mp S_{1P}\mathrm{S} & S_{3P}^2 + \left(1 - S_{3P}^2\right)\mathrm{C} \end{bmatrix} \tag{10}$$

and $C = \cos(\delta z)$ and $S = \sin(\delta z)$.

Matrix (10) describes the case of the most frequently appearing uniform OF with elliptical birefringence without consideration of depolarization and loses. As a result the determination of the MM coefficients by the measurement makes possible to assign the true output SP vector ***S(z)*** to an arbitrary input SP vector ***S(0)***. However the above analytical solution is not applicable to real *non-homogeneous* OF. For the description of a non-homogeneous birefringent fibre a suitable model utilizing the above obtained results for a section of homogeneous OF is needed. Such a model should provide a basis for sufficiently precise numerical solution with acceptable degree of complexity and computation time.

3. The concatenated model of non-homogeneous birefringent optical fibre

One of the possibilities for modelling of nonhomogeneous OF is based on the division of the fibre into a series of sufficiently short sections that can be considered as homogeneous (concatenated model). Each section is characterized by the Mueller matrix $\mathbf{M}_i$ relating the Stokes vectors at the input and output of the i-th section. The basic idea of this model is represented by Fig.1.

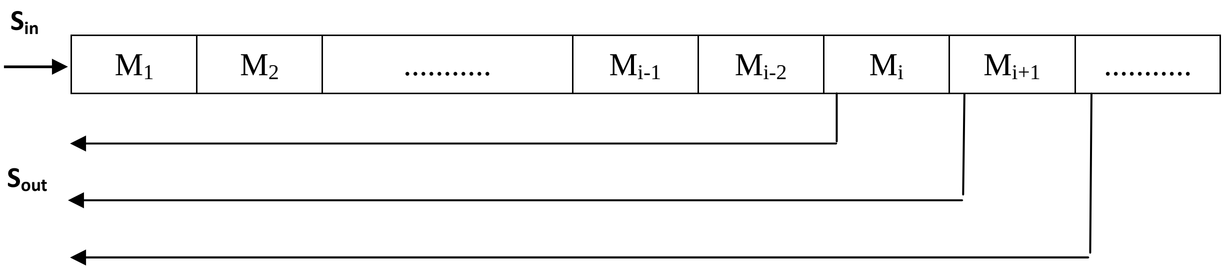

Fig.1. *The basic representation of the „concatenated model" of the non-homogeneous OF*

Originally [9] the basic idea was to divide the whole fibre into a series of equal sections of the length "l_i„ and to describe the relation between the input test impulse SP vector $\boldsymbol{S}_{in}$ and the measured SP vector $\boldsymbol{S}_{out}$ corresponding to the back scattered radiation coming from the n-th section by the following matrix equation

$$\boldsymbol{S}_{out\text{-}i} = (\mathbf{M}_n.\mathbf{M}_{n-1}........\mathbf{M}_2.\mathbf{M}_1)^T.\mathbf{M}_M.(\mathbf{M}_n.\mathbf{M}_{n-1}........\mathbf{M}_2.\mathbf{M}_1).\boldsymbol{S}_{in,} \tag{11}$$

where $\mathbf{M}_i$ is the forward MM of the i-th element and the $\mathbf{M}_M$ is the so called „mirror" MM that changes the „left-handed" SP into „right-handed" one and opposite and the whole first transposed matrix in the brackets of (11) represents the „total backward" MM. The MM of each section can be generally represented by three birefringence parameters – total elliptical birefringence δ_i. azimuthal angle of the fast axes θ_i and the ellipticity angle χ_i [10]. By measuring the $\boldsymbol{S}_{\text{out-i}}$ for a suitable set of $\boldsymbol{S}_{\text{in}}$ and applying the equation (11) for the 1-st section it is possible to calculate directly birefringence parameters of the first section (δ_1, θ_1, χ_1). Then taking together first and second section and having the known parameters of the first section one should get the (δ_2, θ_2, χ_2) of the second section and so on. But this procedure is not easily applicable because as it can be shown the matrix equation for the first section does not provide enough mutually independent equations. This problem can be solved by dividing each elementary section into two equal subsections and then there will be enough independent equations for the calculation of all birefringence parameters of the first section and subsequently of all others. But this "concatenated" procedure is not suitable due to the gradual accumulation of the numerical errors by the transmission of those errors from the first section to the second and then to the third and so on. To avoid that problem one can use the other model based on the equivalent replacement of MM of all elementary sections located before the chosen n-th section by the MM of the equivalent "general elliptic retarder (GER)" as it is described in more detail in the following section.

4. General elliptic retarder model of the non-homogeneous birefringent optical fibre

Let us consider a non-homogeneous twisted optical fibre with the combined linear birefringence "β_L" and the circular birefringence $\rho=\tau(2\text{-}g)$. Circular birefringence is a combination of the geometrical one caused by the physical twist of the fibre "τ" (given in rad/m) and that caused by elasto-optic phenomenon represented by the stress optic coefficient "-g" (g=0,16 for Si OF). Under these conditions for a given section "n" on the fibre (length $l_n=l_1+l_2+l_3$) located at point "z", see Fig.2, it is possible to write the following total MM that describes the forward path (0-z-l_1) and return path (l_1-z-0) polarization transmission properties of the OF under test

$$\boldsymbol{S}_{\text{out-n}}(z) =[\mathbf{M}_{\text{nRET}}(\delta_2, \theta_2, \chi_2)].[\mathbf{M}_{\text{nF}}(\delta_F, \theta_F, \chi_F)].[\mathbf{M}_{\text{nFOR}}.(\delta_1, \theta_1, \chi_1)].\, \boldsymbol{S}_{\text{in}}\,, \qquad (12)$$

where $\mathbf{M}_{\text{nFOR}}.(\delta_1, \theta_1, \chi_1)$, $\mathbf{M}_{\text{nRET}}(\delta_2, \theta_2, \chi_2)$ are the arbitrary MM describing the transmission of the PS corresponding to the forward and backward propagation of the radiation from the source to the begin of the n-th fibre section and back respectively. The relevant n-th optical fibre section under analysis is described by the MM $\mathbf{M}_{\text{nF}}(\delta_F, \theta_F, \chi_F)$ that includes *the forward propagation of test impulse polarization and the return path of the back scattered radiation polarization transmission* from the chosen fibre section with the local PSP parameters (δ_F, θ_F, χ_F). These parameters are functions of the physical or fundamental fibre section parameters "τ", "-g" and "β_L". The section is considered as "*homogeneous*". It was formally approved [10] that "forward-backward MM" of the fibre section $\mathbf{M}_{\text{nF}}(\delta_F, \theta_F, \chi_F)$ can be written in the form

$$\mathbf{M}_{\text{nF}}(\delta_F, \theta_F, \chi_F)]= [\mathbf{M}_{\text{FS}}(\delta_F,\theta_F, \chi_F)]^T.\mathbf{M}_{\text{MIR}}.\, [\mathbf{M}_{\text{FS}}(\delta_F,\theta_F, \chi_F)] \qquad (13)$$

where $\mathbf{M}_{\text{MIR}}$ is a unit diagonal "mirror matrix" with the negative unit in the last row and column that changes the left-handed polarization into right-handed and opposite. Taking into account the meaning of the equation (6) or its equivalent (9) and the MM (10) and also the fact that the n-th fibre section at position „z" is homogeneous it is possible to state that forward MM $\mathbf{M}_{\text{FS}}(\delta_F, \theta_F, \chi_F)$ describes the rotation of the PS vector $\boldsymbol{S}(z)$ around the total birefringence vector of that section that is parallel with the PSP vector $\boldsymbol{S}_P$ defined by (7).

To extract the birefringence parameters of the analysed fibre section (δ_F, θ_F, χ_F) one can use the equation (12). It is necessary to realize that the total transfer MM including the forward path to the n-th section plus forward-return path of the n-th section and backward path to the detector is actually an arbitrary „general elliptic retarder“ characterized by three parameters (δ,θ,χ) and defined by the equation

$$\boldsymbol{S}_{\mathrm{out}}(z) = \mathbf{M}_{\mathrm{GER}}(\delta,\theta,\chi).\boldsymbol{S}_{\mathrm{in}} \tag{14}$$

Generally speaking the unknown GER parameters (δ,θ,χ) can be extracted from the measurement results of the $\boldsymbol{S}_{out}(z)$ for the defined three input SP vectors $\boldsymbol{S}_{in}$. However due to the fact that in the (12) there are 8 unknown parameters ($\delta_1,\theta_1, \chi_1,\delta_2,\theta_2, \chi_2,\beta_L,\tau$) the measurement for the set of three $\boldsymbol{S}_{in}$ does not provide sufficient number of equations for calculation of 8 unknown parameters. However it is possible to solve this problem by dividing the analysed fibre section into three equal parts and perform the measurements of pertaining GER for three points in that section (l_1, l_2, l_3), see Fig.2. But it is possible only in the case if the *divided section is homogeneous* so the parameters (δ_i,θ_i,χ_i) are equal in all subsections. In such a way one shall have enough equations for the unambiguous calculations of all 8 unknown parameters pertaining to the analysed fibre section.

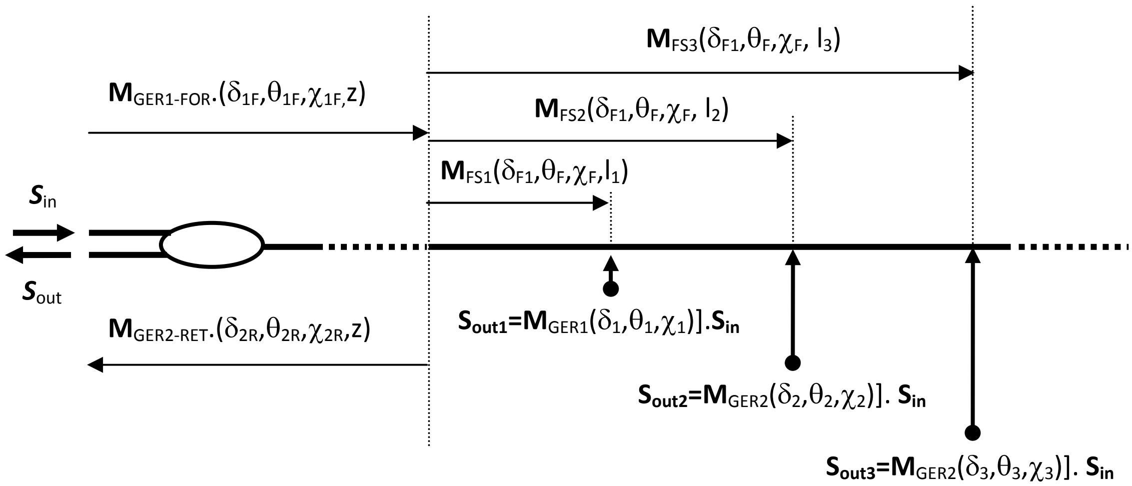

Fig.2. *The relations between the particular parts of GER Mueller matrices*

5. General elliptic retarder model verification

The possibility of the application of the above described ”GER model” for the extraction of the local birefringence parameters of the non-homogeneous OF was verified by numerical simulation using the tools of MATLAB software. The equation (10) represents the MM of a GER in terms of PSP parameters (δ, θ, χ). Therefore it is possible to calculate these parameters for our three MM of total GER describing the relation between the chosen input SP vectors $\boldsymbol{S}_{in}$ and measured output SP vectors $\boldsymbol{S}_{out}$ for a given OF section with known birefringence parameters. The calculated MM parameters actually represent the measured ones. On the other hand we are able to write explicitly the corresponding MM of the total GER for the particular three subsections of the analysed OF section in terms of the sought parameters ($\delta_1,\theta_1, \chi_1$)≡ GER1, ($\delta_2,\theta_2, \chi_2$) ≡GER2, ($\delta_F,\theta_F, \chi_F$) ≡GERF (see Fig. 3). Comparing the (measured) simulated components of the MM for the total GERs of three subsections and the corresponding ones expressed analytically in terms of ($\delta_i,\theta_i, \chi_i$) allows one to write 27 algebraic highly nonlinear equations for unknown parameters ($\delta_i,\theta_i, \chi_i$). The solution of that set of equations is not simple. Therefore one of the suitable approaches to the solution of that problem is the application of the suitable numerical tools of such softwares like Matlab or Mathematica.

In the following (see Table 1) we bring the results of such a numerical simulation using Matlab 12 (tool „fsolve“). The sections/subsections of the length of 90/30 cm on three different OF with a given parameters of elliptic birefringence (δ_{Fi},θ_{Fi},χ_{Fi}, i=1,2,3) were simulated using chosen „arbitrary“ parameters of GER1 and GER2 as it was mentioned in the model description. The results are summarized in the Table 1.

Table 1: *The results of the numerical evaluation of the OF sections (GER $\delta_{Fi}, \theta_{Fi}, \chi_{Fi}$)*

	OF1			**OF2**			**OF3**		
OF section parameters:	δ_1	θ_1	χ_2	δ_2	θ_2	χ_2	δ_3	θ_3	χ_3
Given values:	0,23898	0,61796	0,21816	0.37193	0,65161	0,52359	0.32208	0.11117	0.73303
Calculated values:	0,23898	0,61796	0,22637	0.37193	0,65161	0,54071	0.32208	0.11117	0.74107
Deviation in [%]:	0,00000	0,00000	-3.7636	0.00000	0.00000	-0.7597	0,00000	0,00000	-1.0960

The obtained results characteristic by the maximum error of cca 3 % indicate the feasibility of the used „GER method“ for the extraction of OF birefringence parameters from measured data by PO-OTDR in practical applications like OFSDP.

6. Conclusions

A model for the extraction of the local optical fibre birefringence parameters from the measured data by PO-ODTR method was presented and described. The basic idea consists in the appropriate use of the general elliptic retarder representation for the forward path of the testing impulse optical radiation to the selected section of the measured OF and for the forward-backward path of the selected fibre section and for the backward path of the backscattered radiation. By dividing the analysed section into three parts and under the condition that the local birefringence parameters along the section are constant one can obtain enough equations for the unambiguous calculation (extraction) of the sought local birefringence parameters of the OF section from measured data. The significant advantage of the method is in avoiding the accumulation of numerical errors as it is the case e.g. in the “concatenation model”. On the other hand the necessary numerical solution of a set of rather complicated nonlinear equations may become in the case of application of not enough efficient numerical software tools a critical point of the method. But at present time it seems to be not a serious problem. The method may be applied especially in the design and realization of OFSDP.

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