NEW BEAM FINITE ELEMENT FOR DYNAMIC ANALYSIS OF FUNCTIONALLY GRADED MATERIAL STRUCTURES

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1. Introduction

Mechatronic is one of the most dominant research and application areas in nowadays' engineering, consumer electronics and services. Mechatronic systems represent complex integrated intelligent systems making use of a synergy between information technology, electronics, mechanics, communication and control. For an optimal utilization of their enormous potential it is necessary to examine, analyze, model, control and optimize their structure and parameters for a wide range of applications. Development of dominant mechatronic parts like sensors and actuators still continues and is strongly dependant on the design of new materials and applications of modern approaches from the information, communication and control technologies.

Mechanical resonance of mechatronic devices is in most cases a negative phenomenon. If the frequency of the harmonic loads coincides with the device eigenfrequency, the amplitude of mechanical vibration rises theoretically into infinity, which can lead to device inoperability or destruction. Therefore, the calculation of the device eigenfrequencies is straightforward.

Modeling of structure made by FGM by classical finite elements is very difficult, because each finite element has to have other material properties. The finite element mesh has to be very fine, which makes the preparing of the task input data very time consuming. Therefore, in contribution [1], new 2D FGM beam finite element was developed which is able for modal analysis of the plane beam structures. This new beam finite element will be briefly described in this contribution, and will be used in modal analysis of chosen FGM beam structures.

2. FGM beam finite element

Let us consider a two nodal straight beam element with constant rectangular cross-sectional area $A = bh$ and the quadratic moment of inertia $I = bh^3/12$ (Figure 1). Here, $k(x), k_x(x)$ and $\bar{k}(x)$ is the longitudinally varying transversal, axial and rotational elastic foundation, respectively.

The functionally graded material of this beam arises from mixing two components (formally named as matrix and fibres) that are approximately of the same geometrical form and dimensions. The continuous spatial variation of the effective material properties can be caused by continuous spatial variation of both the volume fraction and material properties of the FGM constituents. Both the fibers volume fraction $v_f(x, y)$ and the matrix volume fraction $v_m(x, y)$ are chosen as polynomial functions of *x*, and with continuous and symmetrical variation through its height *h* with respect to the neutral plane of the beam. The volume fractions are assumed to be constant through the cross-section depth *b*. At each point of the beam it holds: $v_f(x,y)+v_m(x,y)=1$. The values of the volume fractions at the nodal points are denoted by indices *i* and *j*. The material properties of the constituents (fibres - $p_f(x,y)$ and matrix - $p_m(x,y)$) vary analogically (depending on inhomogeneous temperature field for example) as stated by the variation of the volume fractions.

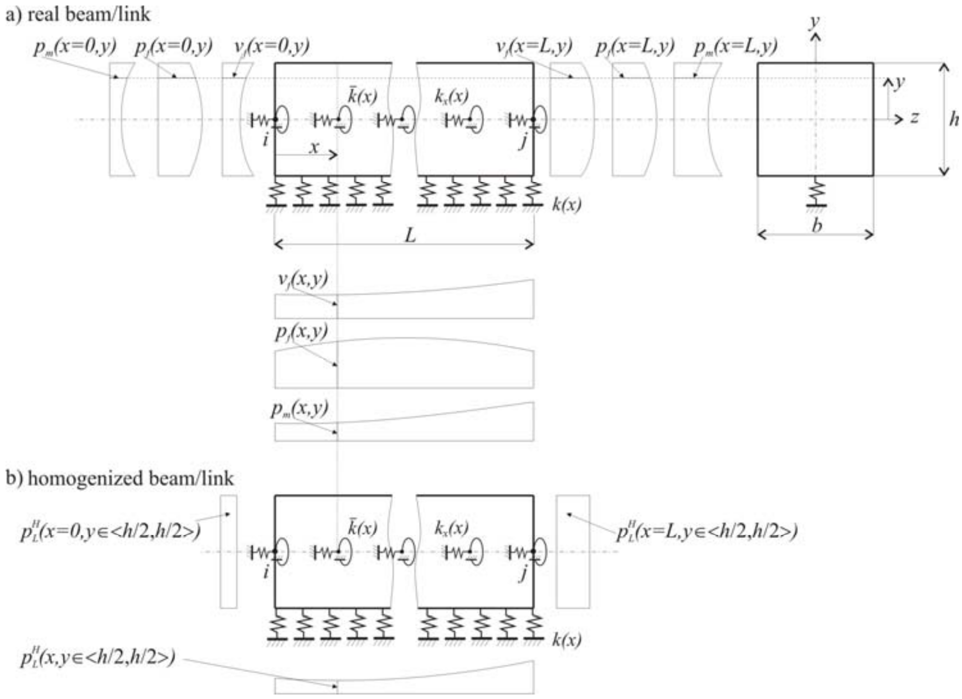

Fig.1: *Real and homogenized FGM beam/link element.*

In the homogenization of the spatial varying material properties the direct integration method will be used [2]. From the assumption that the respective property (e.g. stiffness) of the real beam must be equal to the analogical property of the homogenized beam, the homogenized longitudinal elasticity modules for: tension – compression $E_L^{NH}(x)$, bending $E_L^{MH}(x)$, shear $G_L^H(x)$, and the homogenized mass density $\rho_L^H(x)$ can be calculated, respectively. The effective material properties vary continuously along the local finite element axis *x*. The average shear correction factor k^{sm} has been calculated from the shear stress energy function $k^s(x)$ [2].

The local finite element equation of new 2D beam FGM finite element is:

$$\underbrace{\begin{bmatrix} N_i \\ R_i \\ M_i \\ N_j \\ R_j \\ M_j \end{bmatrix}}_{\mathbf{F}^{loc}} = \underbrace{\begin{bmatrix} B_{1,1} & 0 & 0 & B_{1,4} & 0 & 0 \\ 0 & B_{2,2} & B_{2,3} & 0 & B_{2,5} & B_{2,6} \\ 0 & B_{3,2} & B_{3,3} & 0 & B_{3,5} & B_{3,6} \\ B_{4,1} & 0 & 0 & B_{4,4} & 0 & 0 \\ 0 & B_{5,2} & B_{5,3} & 0 & B_{5,5} & B_{5,6} \\ 0 & B_{6,2} & B_{6,3} & 0 & B_{6,5} & B_{6,6} \end{bmatrix}}_{\mathbf{B}^{loc}} \cdot \underbrace{\begin{bmatrix} u_i \\ w_i \\ \varphi_i \\ u_j \\ w_j \\ \varphi_j \end{bmatrix}}_{\mathbf{U}^{loc}} \tag{1}$$

The non-constant element matrix terms $B_{i,j}$ (functions of natural frequency ω, the 2nd order beam theory axial force N^{II} (it has to be known), the shear correction factor, the varying stiffness and other beam parameters) of the local finite element matrix $\mathbf{B}^{loc}$ are not expressed in detail here from space spending point of view. $\mathbf{F}^{loc}$ and $\mathbf{U}^{loc}$ is the vector of the local forces and vector of the local displacements, respectively. When local matrix $\mathbf{B}^{loc}$ is known the global finite element matrix $\mathbf{B}^{glob}$ (according the global coordinate system) can be derived. The global matrix $\mathbf{B}^{glob}$ is obtained by usual transformation of the local matrix $\mathbf{B}^{loc}$. Derivation of the local finite element equation (1) and its transformation in a global coordinate system is described in [1] in detail.

3. Numerical experiment

The actuator has been considered as the beam structure (shown in Figure 2). It consists of 14 parts - beams. Their square cross-section is constant $b = h = 10\,\mu\text{m}$. Lengths of the parts are: $L_i = 300\,\mu\text{m}$, $i = \{1,2,3,4,5,6,9,10,11,12,13,14\}$ and $L_7 = L_8 = 600\,\mu\text{m}$. The angles α_1 and α_2 are: $\alpha_1 = 70°, \alpha_2 = 20°$. Actuator of such geometry can be used as a rotary microengine [3].

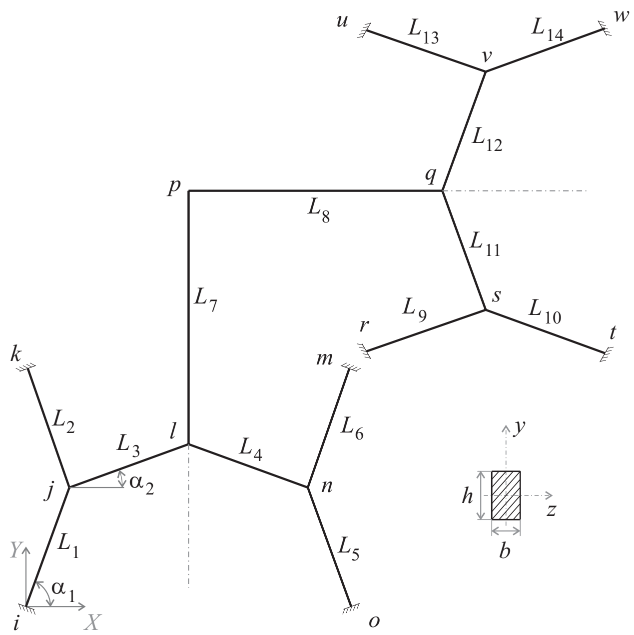

Fig.2: *The geometry of the actuator.*

Material of the beams consists of two components: aluminum Al6061-TO as a matrix and titanium carbide TiC as a fibre. Material properties of the components are constant (not temperature dependent): Al6061-TO - the elasticity modulus $E = 69.0$ GPa, the mass density $\rho = 2700$ kgm^{-3}, the Poisson's ratio $\nu = 0.33$; TiC - the elasticity modulus $E = 480.0$ GPa, the mass density $\rho = 4920$ kgm^{-3}, the Poisson's ratio $\nu = 0.20$.

There are considering two different longitudinal variation of the fibres volume fraction and have been chosen as the polynomial function of the local beam axis *x*:

a) $v_f(x) = 1 - \frac{1}{150}x + \frac{1}{90000}x^2$ b) $v_f(x) = \frac{1}{100}x - \frac{1}{30000}x^2$ c) $v_f(x) = \frac{1}{200}x - \frac{1}{120000}x^2$

The first variation of the fibres volume fraction (denoted by a) has been considered in parts 1, 2, 5, 6, 9, 10, 13 and 14 (with initial point *i*, *k*, *m*, *o*, *r*, *t*, *u* and *w*), the second variation of the fibres volume fraction (denoted by b) in parts 3, 4 11, and 6 and third variation of the fibres volume fraction (denoted by c) in parts 7 and 8. The same values of the fibres volume fraction at the points *j*, *l*, *n*, *p*, *q*, *s* and *v* have been assumed.

The effective material properties of the homogenized beams (as a function of their local *x*-axis) have been calculated by the direct integration method. Because of only longitudinal variation of the constituents volume fraction in this case, the homogenized elasticity modulus (for axial and transversal loading) are equal each other.

The average shear correction factor [2] for all beams is $k^{sm} = 0.8\overline{3}$ (constant Poisson ratio has been assumed for simplicity). The coupled modal analysis of the FGM actuator clamped at the nodes *i*, *k*, *m*, o, *r*, *t*, *u* and *w* has been studied. Only one our new finite element was used for each actuator's part (totally 14 elements). The same problem has been solved using a fine mesh – 2400 of BEAM3 elements (each element has different constant material properties) of the FEM program ANSYS. The average relative difference Δ [%] between eigenfrequencies calculated by our method and the ANSYS solution has been evaluated.

Table 1: *Eigenfrequency of the actuator made of one constituent*

Eigenfrequency [Hz]	New finite element	ANSYS	Δ (%)
1st	135960	138880	2.10
2nd	175544	179060	1.96

As shown in Table 1, the values obtained by both finite elements agree very well with each other.

4. Conclusion

Modal analysis of chosen actuator which is built of the FGM beams has been done by our new 2D beam finite element. Continuous longitudinal variation of material properties has been considered. Shear force effect was considered - the average value k^{sm} of the shear correction function $k^{s}(x)$ has been applied.

The obtained results have been studied and compared with results obtained using a very fine mesh of the FEM program ANSYS. The main additions of our new approach are:

- By new finite element can be very effectively analyzed not only a single beams, but we can analyze the beam structures made of FGMs or other composite material with spatial variation of material properties.
- Our new finite element is very effective and accurate.

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