STIFFNESS OPTIMIZATION OF THE FUNCTIONALLY GRADED MATERIAL MICROACTUATOR

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1. Introduction

The micro-electro-mechanical devices, like the sensors and actuators [1], are very often used in the automotive electronics and mechatronics. The body of the devices is built as the single micro-beam, -plate and -shell as the micro-mechanical structures.

Progress in material engineering plays a significant role in the mechatronic systems design. It is, in particular, the ability to precisely define the local material properties - functionally graded material (FGM) as well as of change properties according to a controlling parameter (usually temperature) - shape memory alloys (SMA). Using new materials like FGM can greatly improved efficiency of a system. The variation of FGM's material properties can be achieved via a controlled uneven mixing of two or more components. From macroscopic point of view, FGM is isotropic in each material point but the material properties can vary continuously or discontinuously in one, two or three directions. The great interest to implement such new materials is in the design of mechatronic parts where it is impossible to change mechanical and other physical attributes through a change in cross-section or complicated geometry.

Using classical finite elements for modeling of FGM structures is very difficult. To capture the variation of material properties the finite element mesh has to be very fine, where each finite element has different constant material properties. Therefore there is the requirement to improved developed existing numerical methods and developed new numerical methods for numerical analysis of such materials.

On the Department of Applied Mechanics and Mechatronics IEAE FEI STU was developed some new FGM beam finite element e.g. for mechanical, coupled electro-thermal-mechanical and modal analysis [2], which will be presented and applied in the following structural analysis of the FGM microactuator.

2. FGM beam finite element

Let us consider a two nodal straight beam element with constant rectangular cross-sectional area $A = bh$ and the quadratic moment of inertia $I = bh^3/12$ (Figure 1).

The functionally graded material of this beam arises from mixing two components (matrix and fibres) that are approximately of the same geometrical form and dimensions. The continuous spatial variation of the effective material properties can be caused by continuous spatial variation of both the volume fraction and material properties of the FGM constituents. The fibers volume fraction $v_f(x, y)$ and the matrix volume fraction $v_m(x, y)$ are chosen as polynomial functions of *x*, with continuous and symmetrical variation through its height *h* with respect to the neutral plane of the beam and constant through the cross-section depth *b*. At each point of the beam it holds: $v_f(x, y) + v_m(x, y) = 1$. The values of the volume fractions at the nodal points are denoted by indices i and j. The material properties of the constituents (fibres - $p_f(x,y)$ and matrix - $p_m(x,y)$) vary analogically (depending on inhomogeneous temperature field for example) as stated by the variation of the volume fractions.

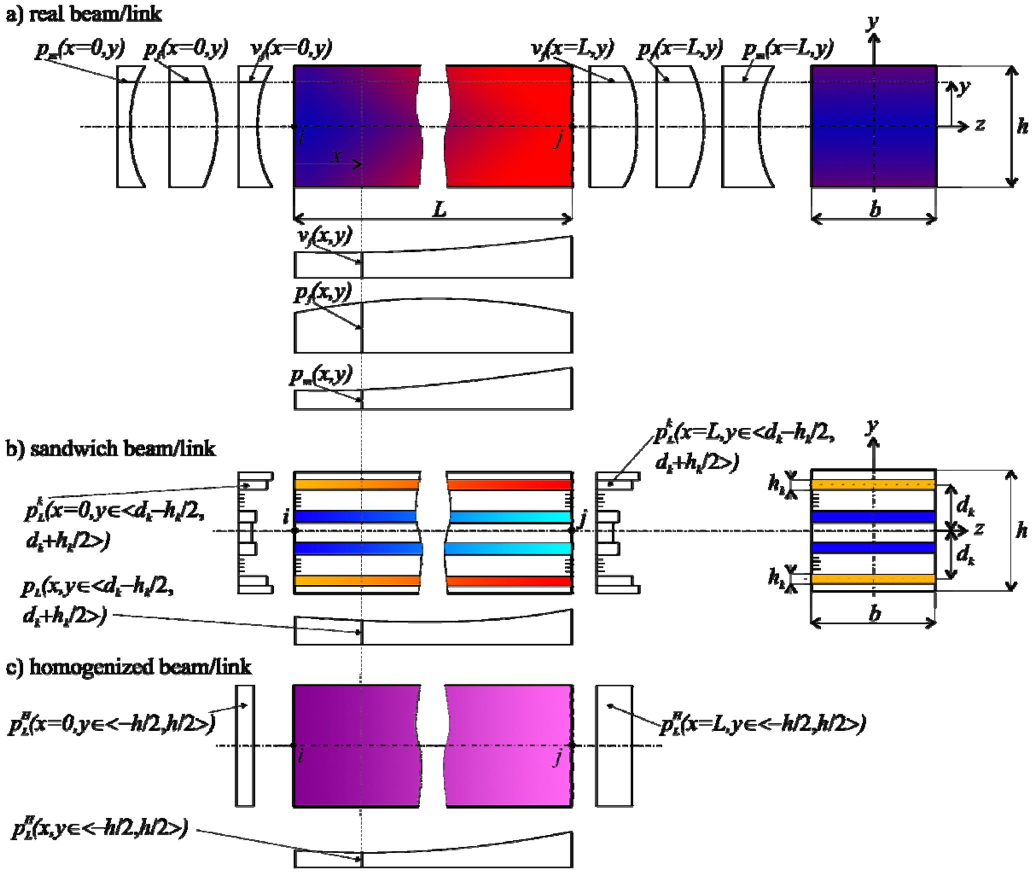

Fig.1: *Real and homogenized FGM beam/link element.*

Homogenization of the varying material properties and the calculation of other parameters can be done using two methods: the layering method (Fig. 1) [3] and the direct integration method [2, 3]. The first one implements extended mixture rules and laminate theory where the real beam has been transformed to a multilayer beam. This method can be also used in the analysis of multilayer beams with discontinuous variation of material properties in transversal direction. In the second one, a direct integration has been used without any division of the beam with continuous transversal variation of material properties into layers.

In the contribution [3] the new 2D FGM beam finite element for modal analysis has been established. Its local finite element equation has a form

$$\underbrace{\begin{bmatrix} N_i \\ R_i \\ M_i \\ N_j \\ R_j \\ M_j \end{bmatrix}}_{\mathbf{F}^{loc}} = \underbrace{\begin{bmatrix} B_{1,1} & 0 & 0 & B_{1,4} & 0 & 0 \\ 0 & B_{2,2} & B_{2,3} & 0 & B_{2,5} & B_{2,6} \\ 0 & B_{3,2} & B_{3,3} & 0 & B_{3,5} & B_{3,6} \\ B_{4,1} & 0 & 0 & B_{4,4} & 0 & 0 \\ 0 & B_{5,2} & B_{5,3} & 0 & B_{5,5} & B_{5,6} \\ 0 & B_{6,2} & B_{6,3} & 0 & B_{6,5} & B_{6,6} \end{bmatrix}}_{\mathbf{B}^{loc}} \cdot \underbrace{\begin{bmatrix} u_i \\ w_i \\ \varphi_i \\ u_j \\ w_j \\ \varphi_j \end{bmatrix}}_{\mathbf{U}^{loc}} \tag{1}$$

$\mathbf{B}^{loc}$ is the local finite element matrix, where its non-constant terms $B_{i,j}$ are functions of natural frequency ω, the 2[nd] order beam theory axial force N^{II}, the shear correction factor,

the varying stiffness and other beam parameters. $\mathbf{F}^{loc}$ and $\mathbf{U}^{loc}$ is the vector of the local forces and vector of the local displacements, respectively.
In this local finite element matrix $\mathbf{B}^{loc}$ the mass and the stiffness matrices are connected together. When the natural frequency ω and the axial force N^{II} are set to zero we get the local stiffness matrix. In this form, this new 2D FGM beam finite can be used for structural analysis of the FGM beam structures.

3. Numerical experiment

The actuator has been considered as the beam structure (shown in Figure 2) that consists of 2 beams. Their square cross-section is constant $b = h = 10\,\mu\text{m}$. Lengths of the parts are: $L_1 = L_2 = 300\,\mu\text{m}$. The angle between the beams is $\theta = 140°$. The actuator is clamped at nodes *i* and *k* and loaded by actuating force $F = 1000\,\mu\text{N}$ at node *j*. The vertical displacement of the point *j* is sought

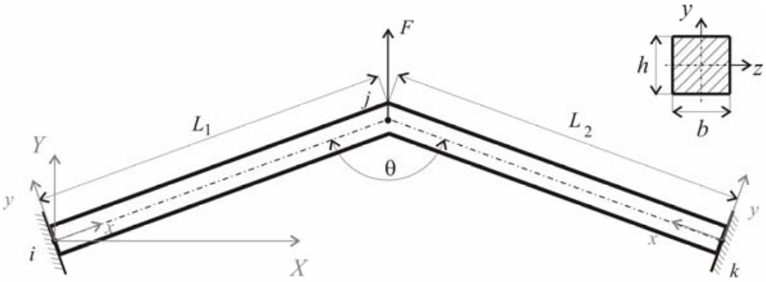

Fig.2: *The geometry of the actuator.*

Three different analyses depending on the type of material have been analyzed in order to finds its influence on the system. In the first analysis the structure is made from constant material (titanium carbide). In the next analyses the actuator is made by FGM (mixing of two components - titanium carbide and aluminum) where different longitudinal variation of material properties has been chosen (Fig. 3).

Material properties of the constituents are constant (not temperature dependent)

aluminum Al6061-TO (use as a matrix)
- elasticity modulus $E = 69.0\,\text{GPa}$
- Poisson's ratio $\nu = 0.33$

titanium carbide TiC: (use as a fibre)
- elasticity modulus $E = 480.0\,\text{GPa}$
- Poisson's ratio $\nu = 0.20$

The longitudinal variation of the fibres volume fraction and have been chosen as the polynomial function (Fig. 3) of the local beam axis *x*:

a) $v_f(x) = 1 - \frac{1}{150}x + \frac{1}{90000}x^2$ b) $v_f(x) = \frac{1}{100}x - \frac{1}{30000}x^2$

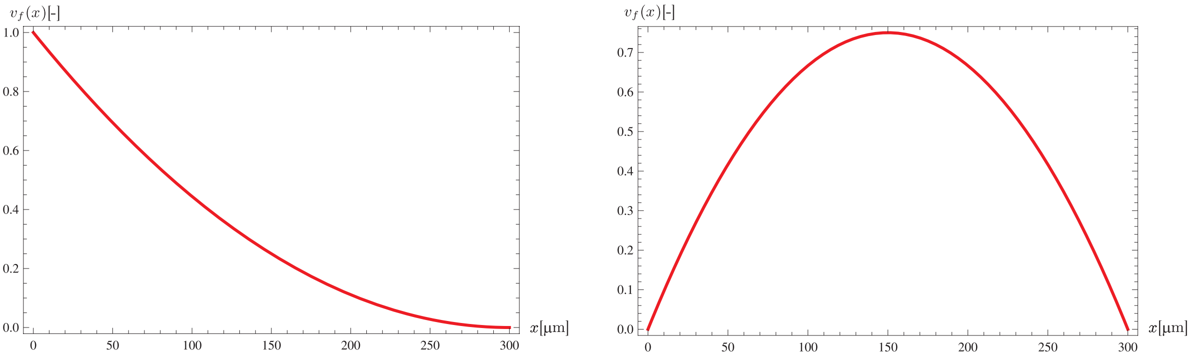

Fig.3: *Fibre volume fraction variation*

The local axis x of the beam 1 begins at node i and ends at point j and the local axis x of the beam 2 begins at node k and ends at point j – see Figure 3. The same values of the fibres volume fraction at the point j have been assumed.

The effective material properties of the homogenized beams (as a function of their local x-axis) have been calculated by the direct integration method. Because of only longitudinal variation of the constituent's volume fraction in this case, the homogenized elasticity modules (for axial and transversal loading) are equal each other.

Only two our new finite element was used – one for each actuator's part. For comparison the same problem has been solved using a fine mesh – 400 of BEAM3 elements (each element has different constant material properties) of the FEM program ANSYS.

Table 1: *Vertical displacement of node j of the actuator*

vertical displacement at point j [μm]	New finite element	ANSYS
constant material TiC	0.0265	0.0265
variation of material properties a)	0.0875	0.0882
variation of material properties b)	0.0558	0.0558

As shown in Table 1, the values obtained by both finite elements agree very well with each other. By mixture of the components the actuating displacement can be change significantly despite the geometry of the actuator was not change.

4. Conclusion

Our new finite element was used for structural analysis of a simple actuator made by FGM beams. This finite element is suitable for analysis not only a single beams but also for effective analyses of mechatronics system (beam structures) made of FGMs with spatial variation of material properties.

The obtained results have been studied and compared with results obtained using a very fine mesh of the FEM program ANSYS. Using different variation of material properties significantly change the behaviour of a system.

Acknowledgement

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