INVESTIGATION OF TECHNOLOGY TEXTURE BY MEASUREMENT OF ELECTRICAL POTENTIAL

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Abstract

Texture is preferred orientation of crystallites in some polycrystalline materials. Different methods are applied to characterize the orientation patterns and determine the orientation distribution. Most of these methods rely on diffraction.

This paper introduces the principle of a method used for characterisation of ceramics texture based on anisotropy of electrical properties of crystallites in ceramics. The mathematical framework of this method is presented in theoretical part of our work. In experimental section we demonstrate how the theoretical result could be used to evaluate technology texture of ceramic material intended for the production of electronic insulators.

1. Introduction

The distribution of crystallographic orientations of a polycrystalline dielectric (texture) has a considerable impact on application properties of the material. The radial texture arising in the manufacture process of cylindrical insulators (technological texture) has a detrimental effect on insulators functionality. It is therefore advisable to check the technological texture of dielectric directly in the production process of insulators.

In general, texture can be investigated by various quantitative techniques (X-ray diffraction, ESBD, SEM, …) and qualitative analysis (polarized microscope, Laue photography, pole figure technique, …). All of the laboratory methods mentioned above are not applicable when we check the technological texture of serial products because they require quite complicated technical equipment and procedures. In our work we introduce the principle of a simple method for the evaluation of radial technological texture in dielectric. The main idea is based on the investigation of electrostatic field radial axial distribution in thin cylindrical samples made from dielectric material.

In the theoretical part we calculate electrostatic field distribution in thin homogeneous cylindrical dielectric sample inserted between electrodes of the capacitor that has circular electrode with unequal diameter. We are especially interested in axially symmetric field dissipation in the area behind the edge of the circular electrode with smaller radius. Generally the typical case of Sturm-Liouville boundary value problem called Bessel`s differential equation arises in calculation of the scalar potential with axial symmetry. We found the particular solution of this equation which defines the Bessel functions and expressed the potential of electrostatic fields in material. Consequently we considered a limit case of extremely slim capacitor and investigated decrease of the field potential in the area behind the edge of smaller electrode by means of asymptotic forms of Bessel functions.

In the experimental part we propose how to use the result of theoretical analysis for the checking of technological texture in practice. Experimental measured radial distribution of electrostatic potential may be different from the theoretical values in some measurement points. The mentioned differences are due to local variations of dielectric properties of material in these points and they allow evaluating the radial technology texture.

2. Theory

We consider a cylindrical dielectric sample with radius R_1 and very small thickness h ($h << R_2$) inserted between two parallel coaxial metal electrodes with radius R_1 and R_2 ($R_2 < R_1$). Voltage U is applied to electrodes (scheme is shown in fig.1).

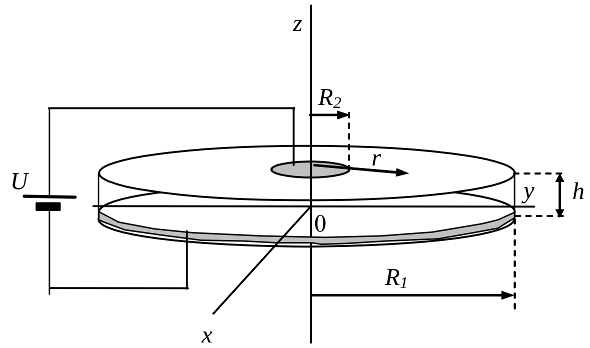

Fig.1

Potential of electrode with radius R_1 is φ_1 and potential of electrode with radius R_2 is φ_2. We are interested in electrostatic field potential φ dependence on distance r from the axis of both plates measured on the sample surface in area $R_2 \le r \le R_1$ (for $z = h$ according fig.1).

2.1 The solution of Laplace equation for axially symmetric electrostatic field

It is possible to consider Laplace equation [1,2] for charges-free electrostatic field in investigation of scalar potential distribution in thin cylindrical dielectric media. The mentioned equation can be written in cylindrical coordinates:

$$\frac{\partial^2 \varphi}{\partial r^2} + \frac{1}{r}\frac{\partial \varphi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \varphi}{\partial \theta} + \frac{\partial^2 \varphi}{\partial z^2} = 0, \qquad (1)$$

If we assume axial symmetry of homogeneous dielectric media, the $\partial^2 \varphi / \partial \theta^2 = 0$ applies. We expect a separable solution:

$$\varphi(r;z) = \Phi(r)\,Y(z). \qquad (3)$$

It results from Laplace equation that components $\Phi(r)$ and $Y(z)$ must obey next equations:

$$\frac{\partial^2 \Phi(r)}{\partial r^2} + \frac{1}{r}\frac{\partial \Phi(r)}{\partial r} = \lambda \Phi(r), \quad \frac{\partial^2 Y(z)}{\partial z^2} = -\lambda Y(z), \qquad (4, 5)$$

where λ is constant. Constant λ can be positive, negative or it can be equal to zero.

a) If $\lambda = 0$ we can easily find the solution of Eq.(1) in the following form:

$$\varphi_0(r;z) = Kz\,ln\,r + L\,ln\,r + Mz + N, \qquad (6)$$

where K, L, M and N are constants of integration depending on boundary conditions.

b) In case if $\lambda > 0$ the solution of Eq.(5) can be found in the form:

$$Y_p(z) = Y_1\,sin(kz + \alpha), \quad \text{where } \lambda = k^2,\ Y_1 = \text{const}, \qquad (7)$$

Eq. (4) can be transformed to a special case of modified Bessel`s differential equation:

$$x^2\frac{\partial^2 \Phi}{\partial x^2}+x\frac{\partial \Phi}{\partial x}-x^2\Phi=0 \ , \quad \text{where } x=kr \, . \tag{8}$$

Next, the solution of Eq.(4) can be determined as follows:

$$\Phi_p(r)=A_2 I_0(kr)+B_2 K_0(kr), \tag{9}$$

where A_2 and B_2 are constants of integration and $I_0(kr)$ and $K_0(kr)$ are modified Bessel`s functions of zero order [3, 4]:

$$I_0(x)=\sum_{m=0}^{\infty}\frac{1}{(m!)^2}\left(\frac{x}{2}\right)^{2m}, K_0(x)=-I_0(x)\left\{\eta+ln\left(\frac{x}{2}\right)\right\}+\frac{2}{1}I_2(x)+\frac{2}{2}I_4(x)+\frac{2}{3}I_6(x)+...$$

c) In case of $\lambda<0$ the solution of Eq.(5) is:

$$Y(x)=A_3 e^{kz}+B_3 e^{-kz} \ , \text{where } \lambda=-k^2 . \tag{10}$$

and A_3 and B_3 are constants of integration. Eq.(4) can be transformed to the next form:

$$x^2\frac{\partial^2 \Phi}{\partial x^2}+x\frac{\partial \Phi}{\partial x}+x^2\Phi=0 \ , \quad \text{where } x=kr \, . \tag{11}$$

Equation (11) is also a special case of Bessel`s differential equation [4, 5] and the solution of Eq.(4) can be written in the form:

$\Phi_p^{(2)}(x)=K_2 I_0(kr)+M_2 Y_0(kr)$, where K_2 and M_2 are constants of integration and:

$$J_0(x)=\sum_{m=0}^{\infty}\frac{(-1)^m}{(m!)^2}\left(\frac{x}{2}\right)^{2m}, \ Y_0(x)=\frac{2}{\pi}\left[J_0(x)\left\{C+ln\left(\frac{x}{2}\right)\right\}+\frac{2}{1}J_2(x)-\frac{2}{2}J_4(x)+\frac{2}{3}J_6(x)+...\right]$$

are Bessel`s functions of zero order. Subsequently, the general solution of Eq.(1) can be found as:

$$\varphi^{(1)}(r;z)=Y^0\left[A_2 I_0(kr)+B_2 K_0(kr)\right] sin(kz+\alpha)+Kz\,ln\,r+L\,ln\,r+Mz+N \ , \text{ if } \lambda>0 \tag{12}$$

$$\varphi^{(2)}(r;z)=\left[K_2 J_0(kr)+M_2 Y_0(kr)\right]\left(A_3 e^{kz}+B_3 e^{-kz}\right)+Kz\,ln\,r+L\,ln\,r+Mz+N \ , \text{ if } \lambda<0 \tag{13}$$

2.1 Application of boundary conditions

Let distribution of scalar potential be determined by function $\varphi^{(I.)}(r;z)$ in area $0\le r\le R_2$, $z\le 0\le h$ (area I). $\varphi^{(I.)}(r;z)$ must obey the following conditions:

$$\varphi^{(I.)}(r;0)=\varphi_1 , \ \varphi^{(I.)}(r;h)=\varphi_2 \ . \tag{14}$$

For this reason, the function $\varphi^{(I.)}(r;z)$ must be written in the form (12) and the following conditions must be satisfied: $sin(kh+\alpha)=0$, $sin(k0+\alpha)=0$. This means $kh+\alpha=n\pi$, $\alpha=m\pi$ (where n and m = 0, ±1, ±2,...). There is no reason for the periodicity of solution $\varphi^{(I.)}(r;z)$. Therefore, it is possible to take into account $\alpha=0$, $k=\pi/h$. Boundary conditions (14) can be satisfied only in case if $K=0$ and $L=0$. For that reason $\varphi^{(I.)}(r;z)$ must be written in the following form:

$$\varphi^{(I.)}(r;z)=\left[A_2\, I_0\left(\frac{\pi}{h}r\right)+B_2\, K_0\left(\frac{\pi}{h}r\right)\right]\sin\left(\frac{\pi}{h}z\right)+M\,z+N \, . \tag{15}$$

Integration constants can be determined by substituting (15) to (14):

$$N=\varphi_1 \ , \quad M=\frac{\varphi_2-\varphi_1}{h}=\frac{U}{h} . \tag{16}$$

and the distribution of scalar potential in this area takes the form:

$$\varphi^{(I.)}(r;z)=\left[A_2 I_0\left(\frac{\pi}{h}r\right)+B_2 K_0\left(\frac{\pi}{h}r\right)\right]\sin\left(\frac{\pi}{h}z\right)+\frac{U}{h}z+\varphi_1 . \qquad (17)$$

Let the distribution of scalar potential be determined by function $\varphi^{(II.)}(r;z)$ in area $R_2 \le r \le R_1$, $z \le 0 \le h$ (area II). Function $\varphi^{(II.)}(r;z)$ must obey the following conditions:

$$\varphi^{(II.)}(r,0)=\varphi_1 \ , \ \varphi^{(II.)}(R_2;z)=\varphi^{(I.)}(R_2;z) \qquad (18)$$

Therefore, function $\varphi^{(II.)}(r;z)$ must be also written in the form (12). The boundary conditions (18) can be satisfied only in case if $L=0$ and $N=\varphi_1$ are considered in Eq.(12).

If we consider the following substitution:

$$M=-K\,ln\,r_0 , \qquad (19)$$

where r_0 is constant, the distribution of scalar potential can be written as follows:

$$\varphi^{(II.)}(r;z)=\left[A_2 I_0\left(\frac{\pi}{h}r\right)+B_2 K_0\left(\frac{\pi}{h}r\right)\right] sin\left(\frac{\pi}{h}z\right)+Kz\,ln\left(\frac{r}{r_0}\right)+\varphi_1 . \qquad (20)$$

We assume that $r \gg h$, then $\pi r/h \gg 1$ and arguments of Bessel`s functions are very large. It holds follows for very large values of arguments in this case [4, 5]:

$$I_0\left(\frac{\pi}{h}r\right)\to\frac{1}{\pi}\sqrt{\frac{h}{2r}}e^{\frac{\pi}{h}r} \ , \ K_0\left(\frac{\pi}{h}r\right)\to\sqrt{\frac{h}{2r}}e^{-\frac{\pi}{h}r}\to 0$$

In addition, the function $\varphi^{(II.)}(r;z)$ must be decreasing and $A_2=0$ for that reason. Consequently, the distribution of scalar potential in thin dielectric media inserted between electrodes is determined by the following functions:

$$\varphi^{(I.)}(r;z)=\frac{U}{h}z+\varphi_1 , \quad \varphi^{(II.)}(r;z)=K\,z\ln\left(\frac{r}{r_0}\right)+\varphi_1 , \qquad (21,22)$$

After substituting Eq.(21) and Eq.(22) to second equation (18) we obtain:

$$K=\frac{U}{h\ln\left(\frac{R_2}{r_0}\right)} \qquad (23)$$

We can find constant r_0 by means of the fact that total electric charge Q accumulated on both electrodes is the same. In the case of thin homogeneous sample we assume:

$$Q=\varepsilon\int_{(S)}\sigma dS=\varepsilon\int_0^{R_2}\sigma^{(I.)}(r)2\pi r dr=\varepsilon\int_0^{R_2}\sigma^{(I.)}(r)2\pi r dr+\varepsilon\int_{R_2}^{R_1}\sigma^{(II.)}(r)2\pi r dr . \qquad (24)$$

where ε is electric permittivity of the material (ε = const in homogeneous case) and $\sigma^{(I.)}$, $\sigma^{(II.)}$ are charge densities on electrodes surface (in area I and area II). The following applies:

$$\sigma^{(I.)}=\varepsilon E_z^{(I.)}(r;0) \ \text{ for } r\le R_2, \qquad \sigma^{(II.)}=\varepsilon E_z^{(II.)}(r;0) \ \text{ for } R_2\le r\le R_1 \qquad (25)$$

where $E_z^{(I.)}$, $E_z^{(II.)}$ are z-components of electrostatic intensity vector in areas I and II respectively. We consider:

$$E_z^{(I.)}=-\frac{\partial\varphi^{(I.)}}{\partial z}=-\frac{U}{h} , \ E_z^{(II.)}=-\frac{\partial\varphi^{(II.)}}{\partial z}=-K_0\ln\left(\frac{r}{r_0}\right) \qquad (26)$$

If we consider Eq.(24) we obtain:

$$\int_{R_1}^{R_2} r\ln\left(\frac{r}{r_0}\right)d\,r=0 . \qquad (27)$$

After integrating of Eq.(27) we can determine r_0 as:

$$r_0 = \frac{R_2}{\sqrt{e}} \left(\frac{R_2}{R_1} \right)^{\frac{R_1^2}{R_2^2 - R_1^2}} \tag{28}$$

and K by substituting of Eq.(28) to Eq.(23) consequently:

$$K = \frac{U}{h\left\{ \left(\frac{R_1^2}{R_2^2 - R_1^2} \right) \ln \frac{R_1}{R_2} + \frac{1}{2} \right\}} \tag{29}$$

We can find the distribution of scalar potential in the area II. on the surface of dielectric media ($z = h$) by substituting Eq.(29) and Eq.(28) to Eq.(22). Next, the voltage measured on the surface of thin sample can be evaluated by:

$$U_v(r;h) = \varphi^{(II.)}(r;h) - \varphi_1 \approx \left\{ 1 - \eta \ln \frac{r}{R_2} \right\} U \ , \quad \text{where} \quad \eta = \frac{1}{\left(\frac{R_1^2}{R_1^2 - R_2^2} \right) \ln \frac{R_1}{R_2} - \frac{1}{2}} \tag{30}$$

3. Experiment

Experimental measurements were carried out on the raw corundum ceramic material. Cylindrical sample with a diameter of 320 mm was prepared. The thickness of sample was 2 mm and applied voltage U = 1.5 V. The voltage U_v was measured in eight directions (I., II.,...VIII) which were rotated by 45°. Measurements were carried out on the surface of the sample by means of equipment illustrated in Fig.1. Radiuses of electrodes were R_1 =8,5 mm and R_2 = 160 mm. Experimental data were compared with the theoretical result (30). The obtained result is shown in Fig.2.

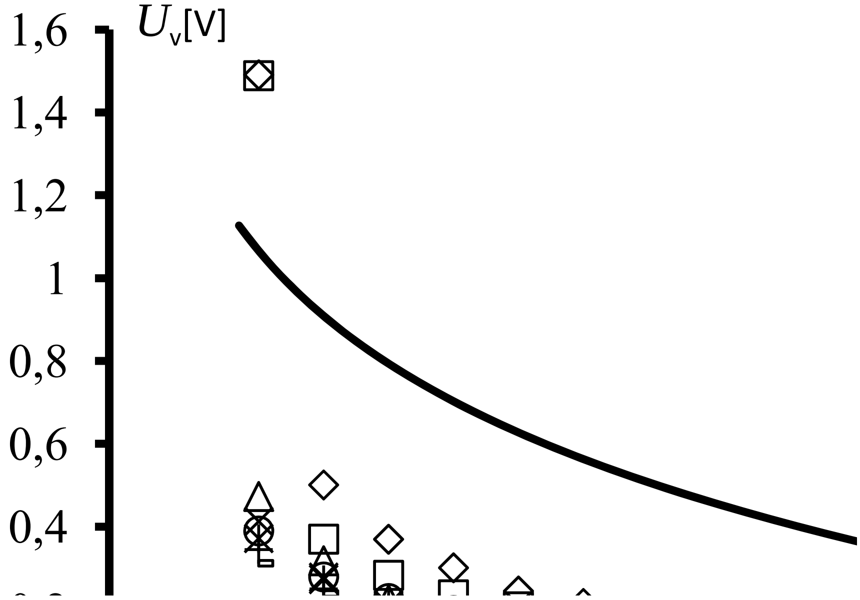

Fig.2 *Discrete symbols - data measured in different directions along diameter of the cylindrical sample. Solid line - theoretical result*

4. Conclusion

The quality and efficiency of the technology of electronic insulators for very high voltage (VHV) depend on the characteristics of the used ceramic material. Many authors have dealt with the evaluation of the texture and dielectric properties of materials in the past [6-12]. In

our work we scanned local changes of dielectric properties of a thin ceramic sample by means of the measurement of distribution of scalar electric potential on the sample surface. The observed discrepancies between theoretical and measured values are caused by the local inhomogeneities in material structure. It should be noted that our theoretical result (30) was obtained for the homogeneous sample under the simplifying assumptions. However, the tendency of dependence (30) is significant in practical use.
In view of the observed homogeneity of the blank (untreated material) structure, the measurement results show a high degree of orientation of particles of the ceramic mass plastic components across the blank. We assume that the observed deviations in electrical potential values are an expression of rearranging of the texture-making particles. The created texture and homogeneous environment in the volume of the blank should lead to the low reject of the produced insulators due to the presence of cracks or reducing their mechanical strength. This assumption was confirmed in technology of insulators production. The manufactured series of insulators were characterized by low levels of the reject. The same conclusion was also obtained by method of an analysis of dimensional changes of the samples before and after firing, which were collected from the profile of the blank from the directions in which the values of the electric potential were measured.

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