ELECTRIC -THERMAL ANALYSIS OF FGM CONDUCTOR JOINT USING NEW FINITE ELEMENT

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1. Introduction

Solution of the thermal-electric problems is very important not only in the design of power lines and electric circuits but also in the analysis of the micro-electro-mechanical systems (MEMS), mechatronic-, and electronic devices. Several electric-thermal finite elements have been developed and implemented into commercial finite element codes, e.g. [1]. Using a very fine mesh of these elements allows also modeling and simulation of devices made of composites and functionally graded materials (FGMs) with inhomogeneous material properties. But preparing of the input data for analysis of these structures is very time consuming, and the results accuracy depends very strongly on the mesh fineness. Development of new finite elements for modeling and simulation of the electric-thermal problems made of FGM with spatial variation of material properties is straightforward.

2. FEM equations of the electric – thermal link

The FEM equations of the homogenized (homogenization method is described in [2, 3, 6] in detail) electric-thermal link with longitudinal variation of the effective electric and thermal conductance have been presented in [2]. After minimization of the potential energy functional [4], the FEM equations for electric conduction can be obtained

$$k_V \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} V_i \\ V_j \end{bmatrix} = \begin{bmatrix} I_i \\ I_j \end{bmatrix} \tag{1}$$

where $k_V = \gamma_{Li}^H A / b'_{2\gamma_L^H}$ is the electric conductivity, V_i, V_j are the nodal electric potentials and I_i, I_j are the nodal currents. Quantity $b'_{2\gamma_L^H}$ is the transfer constant which represents a value of the transfer function $b'_{2\gamma_L^H}(x)$ (Eq. 2) at location $x = L$.

$$b'_{2\gamma_L^H}(x) = \int_0^x \frac{1}{\eta_{\gamma_L^H}(x)} dx \tag{2}$$

Electric potential at location *x* can be expressed as

$$V(x) = \left(1 - \frac{b'_{2\gamma_L^H}(x)}{b'_{2\gamma_L^H}}\right) V_i + \left(\frac{b'_{2\gamma_L^H}(x)}{b'_{2\gamma_L^H}}\right) V_j \tag{3}$$

The FEM equations for thermal conduction including the Joule heat and heat convection are

$$k_T \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} T_i \\ T_j \end{bmatrix} = \begin{bmatrix} P_i \\ P_j \end{bmatrix} + \begin{bmatrix} P_i^J \\ P_j^J \end{bmatrix} \tag{4}$$

where $k_T = \lambda_{Li}^H A / b'_{2\lambda_L^H}$ is the thermal conductivity, T_i, T_j are the nodal temperatures and P_i, P_j are the nodal heat flows. P_i^J, P_j^J are the Joule heats obtained by transformation of generated heat to the nodal points. All these quantities are described in [6] in detail.
The transfer function of heat conduction $b'_{2\lambda_L^H}(x)$ can be expressed analogically to (2). If the effect of distributed heat loads along the link element will be included, the resultant temperature at location x can be expressed as [3]

$$T(x) = \left(1 - \frac{b'_{2\lambda_L^H}(x)}{b'_{2\lambda_L^H}}\right) T_i + \left(\frac{b'_{2\lambda_L^H}(x)}{b'_{2\lambda_L^H}}\right) T_j - \left(\frac{b'_{2\lambda_L^H}(x)}{b'_{2\lambda_L^H}}\right) T_L^J(L) + T_L^J(x) \tag{5}$$

where $T_L^J(x)$ represents the temperature rise due to distributed Joule heat to location x (described in [6]) and $T_L^J(L)$ is its value for $x = L$.

Calculation of the secondary variables is described in [6] in detail.

3. Numerical experiment

A composite conductor joint has been considered as shown in Fig. 1. It consists of three parts – Link 1, Link 2 and Link 3. Their circular cross-section is constant with diameter $d_1 = d_2 = d_3 = 0.01$ m. Lengths of parts are: $L_1 = 0.1$ m, $L_2 = 0.07$ m and $L_3 = 0.05$ m. Material of the links consists of two components: NiFe – matrix (denoted with index *m*); Tungsten – fibre (denoted with index *f*).

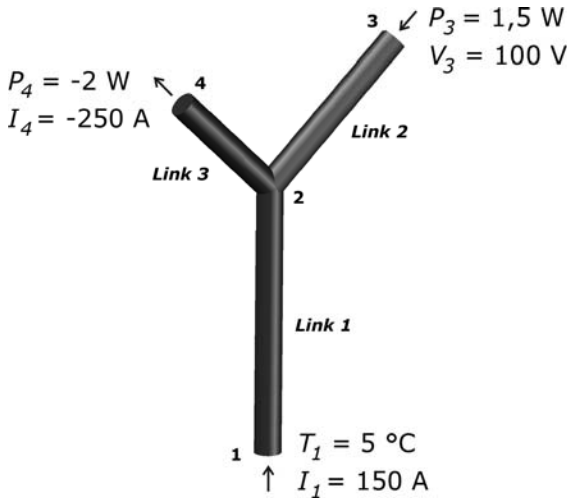

Fig.1: *Composite conductor with spatial variation of material properties: geometry, constrains and loads*

Material properties of the components are constant and their values are: Tungsten (fibres) – thermal conductance λ_f =160W/mK, electric conductance $\gamma_f = 2.84 \times 10^7$ S/m; NiFe (matrix) – thermal conductance λ_m = 100 W/mK, electric conductance $\gamma_m = 1.31 \times 10^7$ S/m [5]. The fibre volume fraction at each link varies linearly and symmetrically according to rotational symmetry axis in the radial direction at node *i* (*j*) and continuously linear in the longitudinal direction to the constant value at node *j* (*i*).

Link 1: node *i* (point 1) $v_{fi} \in$ <0.0, 1.0>, node *j* (point 2) v_{fj} = 0.3. The value v_{fi} = 0.0 is on the link longitudinal axis and v_{fi} =1.0 on the link surface.

Link 2: node i (point 2) v_{fi} = 0.3, node j (point 3) $v_{fj} \in$ <0.5, 1.0>. The value v_{fj} = 0.5 is on the link longitudinal axis and v_{fj} =1.0 on the link surface.

Link 3: node i (point 2) v_{fi} = 0.3, node j (point 4) $v_{fj} \in$ <0.5, 1.0>). The value v_{fj} = 1.0 is on the link longitudinal axis and v_{fj} =0.5 on the link surface.

The electric and thermal field variables have been found. Only three our link elements have been used for solution of the above described problem (one for each link). For comparison, the same problem has been solved with very fine mesh (4400 elements) of the LINK68 elements using program ANSYS [1].

The effective longitudinal electric and thermal conductance of the layers for each bar can be calculated according to [6].

The applied constrains and loads are (Fig. 1):

- electric current and potential: I_1 = 150 A, I_4 = -250 A, V_3 = 100 V;
- heat flow and temperature: P_3 = 1.5 W, P_4 = -2 W, T_1 = 5 °C.

The global FEM equations system (6) for electric conduction of this joint was obtained using the equations (3) for each link:

$$\begin{bmatrix} k_{V1} & -k_{V1} & 0 & 0 \\ -k_{V1} & k_{V1}+k_{V2}+k_{V3} & -k_{V2} & -k_{V3} \\ 0 & -k_{V2} & k_{V2} & 0 \\ 0 & -k_{V3} & 0 & k_{V3} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} \tag{6}$$

By solving the equations (6) the nodal electric variables have been obtained: V_1 =100.0065 V; V_2 = 99.9958 V; V_4 = 99.9880 V; I_3 = 100 A.

The total FEM equations system (7) for thermal conduction has been obtained by the similar way as the equations (6).

$$\begin{bmatrix} k_{T1} & -k_{T1} & 0 & 0 \\ -k_{T1} & k_{T1}+k_{T2}+k_{T3} & -k_{T2} & -k_{T3} \\ 0 & -k_{T2} & k_{T2} & 0 \\ 0 & -k_{T3} & 0 & k_{T3} \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{bmatrix} = \begin{bmatrix} P_1 + P_{11}^J \\ P_2 + P_{12}^J + P_{22}^J + P_{32}^J \\ P_3 + P_{23}^J \\ P_4 + P_{34}^J \end{bmatrix} \tag{7}$$

Longitudinal distribution of the temperature (5) for n = 20 layers in each link is shown in Fig. 2. By solving these equations the nodal variables have been obtained: T_2 = 33.54 °C; T_3 = 44.95 °C; T_4 = 28.45 °C; P_1 = - 3.46 W.

The results comparison of the nodal electric variables is shown in Table 1 and the nodal thermal variables in Table 2.

Tab. 1. *Comparison of the nodal electric variables.*

n	V_1 [V]	V_2 [V]	V_4 [V]	I_3 [A]
20	100.00649	99.99586	99.98804	100
ANSYS	100.00649	99.99586	99.98804	100

Tab. 2. *Comparison of the nodal thermal variables.*

n	T_2 [°C]	T_3 [°C]	T_4 [°C]	P_1 [W]
20	33.5397	44.9476	28.4468	-3.46409
ANSYS	33.5388	44.9164	28.3929	-3.46369

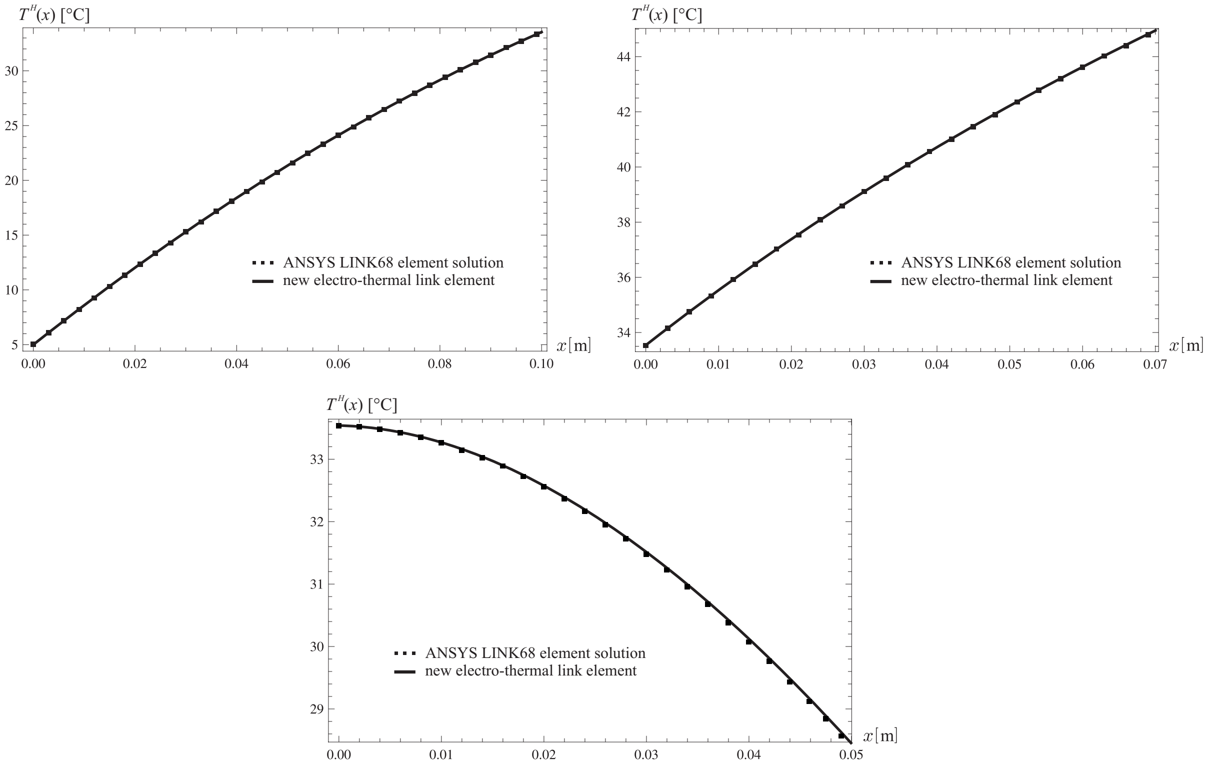

Fig.2: *Longitudinal distribution of the temperature in the link 1, 2 and 3.*

4. Conclusion

Comparison of the solution results for conductor joint obtained by only three our new FGM electro-thermal link finite elements (one for each link) with the very fine mesh of the solid finite elements shows very high effectiveness and accuracy (Fig. 2) of our solution method. Our new link element is very effective and accurate in analysis of electric circuits with long and thin conductors where using of the 2D or 3D finite elements would be much complicated and purposeless in the mesh fineness and input data preparing point of view.

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